# **Critical Early Numeracy Outcomes:** Addressing access to foundational mathematics in low attainment contexts

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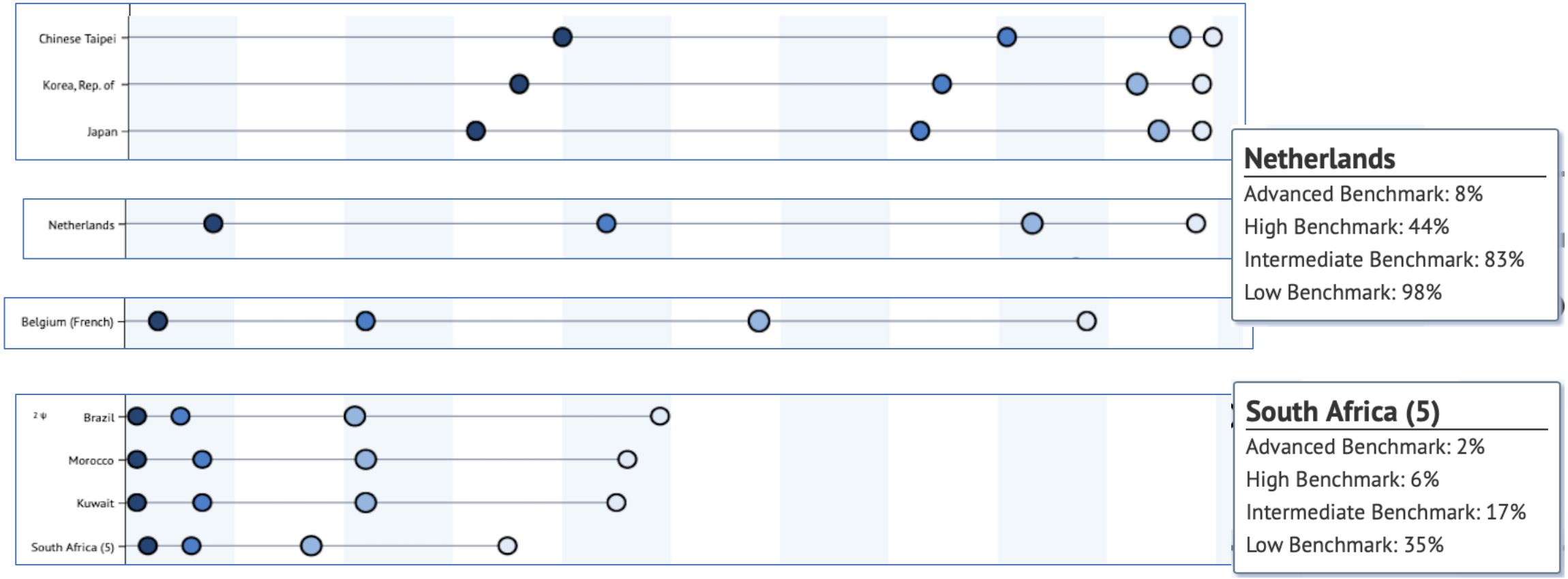
# Low attainment in mathematics

- Around the world, ongoing concern with low attainment in mathematics.
- Concerns often framed in economic terms, for individuals and for society

- But while the focus on low attainment is broadly international, the scale of concern varies in different parts of the globe.

# Low attainment in mathematics

TIMSS 2023: Mathematics Gr4
International benchmarks offer initial insights into differences



# What is the Low International Benchmark?

*Students demonstrate basic mathematical understanding.*

They can add and subtract whole numbers with up to three digits, multiply and divide single-digit whole numbers, and solve simple word problems.

They can apply basic measurement ideas and properties of common geometric shapes. Students can read data from different representations and complete simple bar graphs.

von Davier, M., Kennedy, A., Reynolds, K., Fishbein, B., Khorramdel, L., Aldrich, C., Bookbinder, A., Bezirhan, U., & Yin, L. (2024). *TIMSS 2023 International Results in Mathematics and Science*. Boston College, TIMSS & PIRLS International Study Center. https://doi.org/10.6017/lse.tpisc.timss.rs6460

# UNESCO evidence

UNESCO surveys suggest even broader concerns:

- ‘pre-pandemic data from 34 low- and middle-income countries featured in the report shows that … three quarters of schoolchildren in Grade 4 fail to obtain foundational numeracy skills’
- ‘Pre-pandemic data from 79 middle- and high-income countries show more than a third of 15-year-old schoolchildren have yet to grasp minimum proficiency in mathematics’

The majority of the world’s children are not meeting very basic levels of understanding in early mathematics

UNESCO (2022). Solving the equation: Helping girls and boys learn mathematics.
https://www.unicef.org/media/127261/file/Solving%20the%20Equation.pdf

# TIMSS influence on curricula

- Evidence of curricular ‘convergence’ internationally around topics and skills to cover, and emphasis on problem-solving and reasoning[1]
- International ‘benchmarking’ processes on curricular development, increasingly common

But evidence indicates that ambitious curricula are poorly aligned with learning levels of majority of children in many countries[2].

[1]Kadijevich, D.M., Stephens, M., Solares-Rojas, A., Guberman, R. (2023). Impacts of TIMSS and PISA on Mathematics Curriculum Reforms. In: Shimizu, Y., Vithal, R. (eds) Mathematics Curriculum Reforms Around the World. New ICMI Study Series. Springer, Cham. https://doi.org/10.1007/978-3-031-13548-4_22

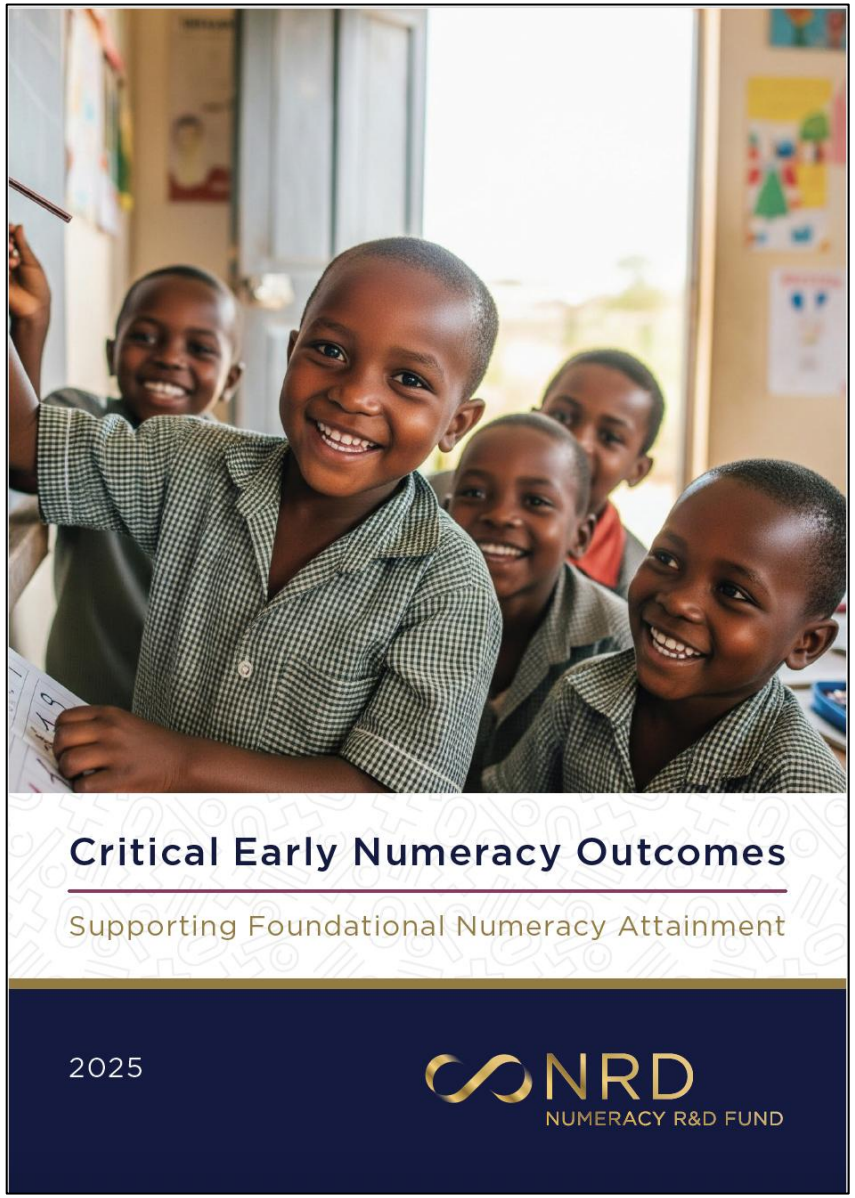
[2]Pritchett, L. & Beatty, A. (2015). Slow down, you’re going too fast. International Journal of Educational Development, 40. https://doi.org/10.1016/j.ijedudev.2014.11.013

# How to intervene?

- Considering a subset of early numeracy outcomes that are critical to ensure learning of

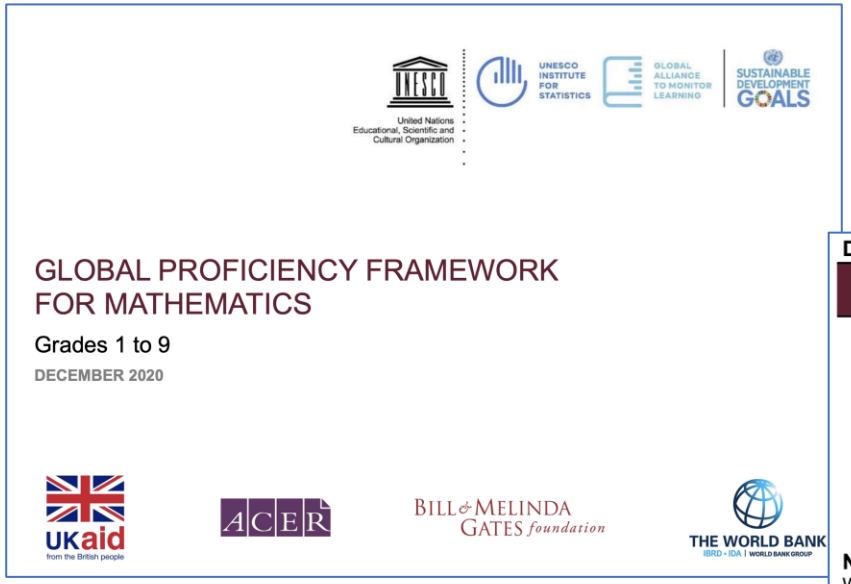
FOR

- Any possibility of access to mathematics beyond the early grades




https://nrdfund.org/wp-content/uploads/2026/03/GENESIS_NRD-Critical-Early-Numeracy-Outcomes-2025_Final-1.pdf

# Worked from existing specifications used in LMICs




Specifications developed for 'minimum proficiencies' and beyond for design of assessments that can be used to compare outcomes across LMICs

Not intended to support teaching or progressions in learning.

**DOMAIN: N—NUMBER AND OPERATIONS**

| Construct | Subconstruct | Knowledge or Skill | Grade 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| N1 Whole numbers | N1.1 Identify and count in whole numbers, and identify their relative magnitude | N1.1.1 Count, read, and write whole numbers | x | x | x | x | x | x | | | |
| | | N1.1.2 Compare and order whole numbers | x | x | x | x | x | x | | | |
| | | N1.1.3 Skip count forwards or backwards | | x | x | x | x | x | | | |
| | N1.2 Represent whole numbers in equivalent ways | N1.2.1 Determine or identify the equivalency between whole numbers represented as objects, pictures, and numerals | x | x | x | | | | | | |
| | | N1.2.2 Use place-value concepts | | x | x | x | x | x | | | |
| | | N1.2.3 Round whole numbers | | | | x | x | x | | | |
| | N1.3 Solve operations using whole numbers | N1.3.1 Add, subtract, multiply and divide whole numbers | x | x | x | x | x | x | | | |
| | | N1.3.2 Find the double or half of a set of objects | x | x | | | | | | | |
| | | N1.3.3 - Multiply and divide whole numbers | | | x | x | x | x | | | |
| | | N1.3.4 Demonstrate fluency with basic addition and subtraction facts | | | x | x | | | | | |
| | | N1.3.5 Demonstrate fluency with basic multiplication and division facts | | | | x | | | | | |
| | | N1.3.6 Identify factors and multiples of whole numbers | | | | | | x | | | |
| | | N1.3.7 Perform calculations involving two or more operations on whole numbers | | x | x | x | x | x | | | |
| | N1.4 Solve real-world problems involving whole numbers | N1.4.1 Solve real-world problems involving the addition and subtraction of whole numbers, including with measurement and currency units | x | x | x | x | x | x | | | |
| | | N1.4.2 Solve real-world problems involving the multiplication and division of whole numbers, including with measurement and currency units | | | | x | x | x | | | |
| N2 Fractions | N2.1 Identify and represent fractions using objects, pictures, and symbols, and identify relative magnitude | N2.1.1 Express a visual representation of a fraction (picture, objects) in fractional notation | | | x | x | | | | | |
| | | N2.1.2 Identify equivalent fractions | | | | x | x | x | x | | |
| | | N2.1.3 - Identify and express equivalences between improper fractions and mixed numbers | | | | | x | x | x | | |
| | | N2.1.4 - Compare and order fractions and mixed numbers, including when they are positive and negative | | | | x | x | x | x | | |
| | N2.2 Solve operations using fractions | N2.2.1 Add and subtract fractions and mixed numbers | | | | x | x | x | x | | |
| | | N2.2.2 Multiply and divide fractions by whole numbers, fractions, and mixed numbers | | | | | x | x | x | | |
| | N2.3 Solve real-world problems involving fractions | N2.3.1 Solve real-world problems involving the addition and subtraction of fractions (proper and improper), whole numbers, and mixed numbers | | | | x | x | x | x | | |
| | | N2.3.2 - Solve real-world problems involving the multiplication and division of fractions (proper and improper), whole numbers, and mixed numbers | | | | | x | x | x | | |

# What should a critical core consist of?

| Fundamentals of whole number | |
|---|---|
| **Addition and subtraction** | **Multiplication and division** |

What does this mean in terms of progression:

- Within each early grade?
- Across these grades?

What are the dangers of presenting a core sub-set?

What can we do to mitigate the dangers?

# Building research-informed materials that are accessible to teachers and programme developers

- Research findings, not always easily accessible
- Limited time for lengthy reading
- Written in an academic language that can create further barriers:

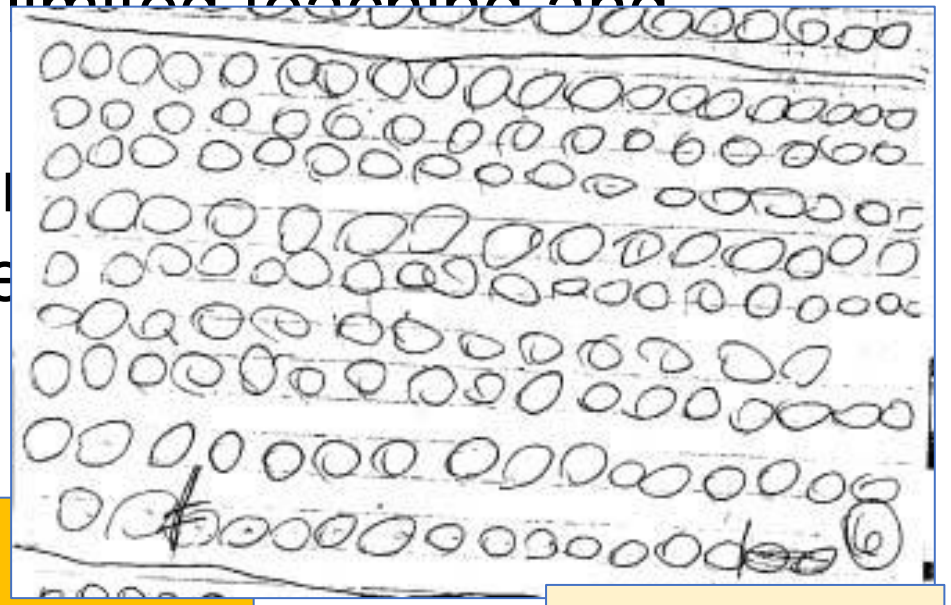
  *‘researchers might seek out training opportunities to develop the skills necessary to communicate their results in a digestible fashion’*

Need ‘translational’ research communication

Barrett, C. A., & Prendergast, M. (2025). A qualitative examination of barriers to disseminating and accessing research in Irish schools. *Irish Educational Studies*, 1–24. https://doi.org/10.1080/03323315.2025.2479447

# LMIC contexts:

- Poor maternal health and early childhood stunting
- Large classes, poor infrastructure, limited teaching and learning resources
- Disruptions to schooling; teacher a
- Gaps in teacher content knowledge teacher professional development



G3: 214 + 12
Hoadley, 2007

In classrooms:
- Chorused responses to closed questions
- Evidence of problems with coherence in instruction
- Limited progression moves

# Numeracy Research and Development (NRD) programme

## What We Are Trying to Solve

Too many children in Africa go to school every day but never truly learn mathematics. They advance through the early grades without mastering the basic numeracy skills that unlock all other learning. This learning gap limits their confidence, curiosity, and their future opportunities. The NRD Fund exists to change that; by supporting African-led research and innovation that helps teachers, schools, and systems ensure every child builds a strong foundation in mathematics from the very start.

## Our Mission

The Numeracy Research & Development (NRD) Fund is a bold new initiative working to improve foundational numeracy outcomes for children in Sub-Saharan Africa. We focus on evidence based solutions that tackle gaps in how numeracy is taught and understood, especially in early primary years. Our goal: To support local governments and partners in helping at least 50% of students in government schools meet Grade 3 numeracy benchmarks after three years of support.

https://nrdfund.org

# Critical Early Numeracy Outcomes: Model

**Fluencies**

Results that most children should know at the level of rapid recall

**Problem-solving**

Setting up, describing and using models of problem situations, moving between representations and working with increasingly efficient calculation strategies.

| Level 1 | Level 2 |
| --- | --- |
| Simpler problems<br>More concrete representations<br>Inefficient counting strategies<br>Simpler language | More complex problems<br>More abstract representations<br>Efficient calculation strategies<br>Expanded language |

## ADDITION & SUBTRACTION GRADE 3 (A&S1)

### Fluency

**Children can:**

- Read, write, count, compare and order numbers up to 1000
- Add/subtract 1-5, 10, and multiples of 10 to/from numbers in the 1-100 range
- Recall addition and subtraction facts in 1-20 range
- Compose/decompose numbers to 1000 using place value
- State the multiple of 10 before/after any number and the nearest 10 for any number in the 1-100 range
- State doubles of 1-10 and multiples of 10
- State halves of even numbers to 20 and multiples of 10

# Critical Early Numeracy Outcomes: Content

## Problem Solving

**Children can work with addition and subtraction in the 1-100 range**

### Level 1

**Children can:**

- Solve problems when the result is unknown (24 + 15 = _ or 83 - 27 = _ )
- Draw diagrams/write number sentences/ work with representations of addition/ subtraction (that include images, part-whole diagrams and number lines)
- Solve addition/subtraction problems using known facts and place value patterns
- Interpret when to add and subtract from simple language statements (e.g. What is the sum of 65 and 17? Or 78 minus 23)

### Level 2

**Children can build on Level 1 learning to:**

- Solve problems when the change/start is unknown (23 + _ = 35 or _ - 16 = 65)
- Draw diagrams/write number sentences/ work with representations of addition/ subtraction (that include part-whole diagrams and number lines)
- Solve addition/subtraction problems using known and derived facts, and use number properties and place value patterns flexibly and efficiently
- Interpret when to add and subtract from more complex language statements (e.g. How much bigger is 47 than 29? Or 45 is 18 less than ?)

**Addition and Subtraction: G3**

# Grade 1 Addition and Subtraction: Problem-solving

## Problem Solving

**Children can work with addition and subtraction in the 1-10 range**

### Level 1

**Children can:**

- Solve problems when the result is unknown (3 + 4 = _ )
- Make/draw/work with representations of addition/subtraction (that include number tracks and ten frames)
- Solve addition/subtraction problems by unit counting (e.g. solve 4 + 3 by counting out 4 fingers on one hand, 3 fingers on the other hand, and then counting all fingers to get 7)
- Interpret when to add and subtract from simple language statements (e.g. add 4 and 3 or 7 take away 3)

### Level 2

**Children can build on Level 1 learning to:**

- Solve problems when the change/ start is unknown (3 + _ = 7)
- Make/draw/work with representations of addition/subtraction (that include number lines and bar diagrams)
- Solve addition/subtraction problems using known and derived facts (e.g. solve 4 + 3 as 2 more than 5 using 4 + 1 = 5 as a known fact)
- Interpret when to add and subtract from more formal or more complex language statements (e.g. How many more is 7 than 4? or subtract 3 from 8)

# ‘Back-fill’ to earlier grades

Grade 1 Addition and Subtraction: Fluencies

## Fluency

**Children can:**

- Subitize numbers
- Read, write, count, compare and order numbers up to 20
- Add/subtract 1 to/from numbers up to 20
- Rapidly recall addition and subtraction facts in 1-5 range and bonds of 10
- State doubles of 1-5

# Grade 3: Multiplication and division

## Fluency

**Children can:**

- Recall multiplication and division facts for 2, 5, and 10 up to 20, 50 and 100 respectively

## Problem Solving

**Children can work with multiplication/division situations presented in pictures and number sentences that use formal language**

**These situations involve multiplying/dividing involving 1x1 to 10x10**

### Level 1

**Children can:**

- Connect number sentences, diagrams and stories of multiplication/division situations including word problems, array and number line images
- Work out products or quotients in number sentences using various strategies such as drawing, repeated addition, known facts, doubling and halving, commutativity

### Level 2

**Children can build on Level 1 learning to:**

- Connect number sentences, diagrams and stories of multiplication/division situations including word problems, array and number line images. Situations may include varied placement of the unknown (e.g. Some friends share 18 candies. If each of them get 3 candies, how many friends were there?)
- Work out missing numbers in number sentences, using various strategies for efficient calculation, including known and derived multiplication facts, doubling and halving, commutativity. (E.g. _ x 3 = 12)

# Dangers in creating a CENO core

- That this becomes the de facto curriculum for teaching, leaving out Shape & Space, and Data Handling
- The fluencies are seen as precursors to problem-solving, with emphasis on disconnected ‘drill’ learning
- That focusing on ‘small steps’ makes more complex problem-solving difficult to reach and attain

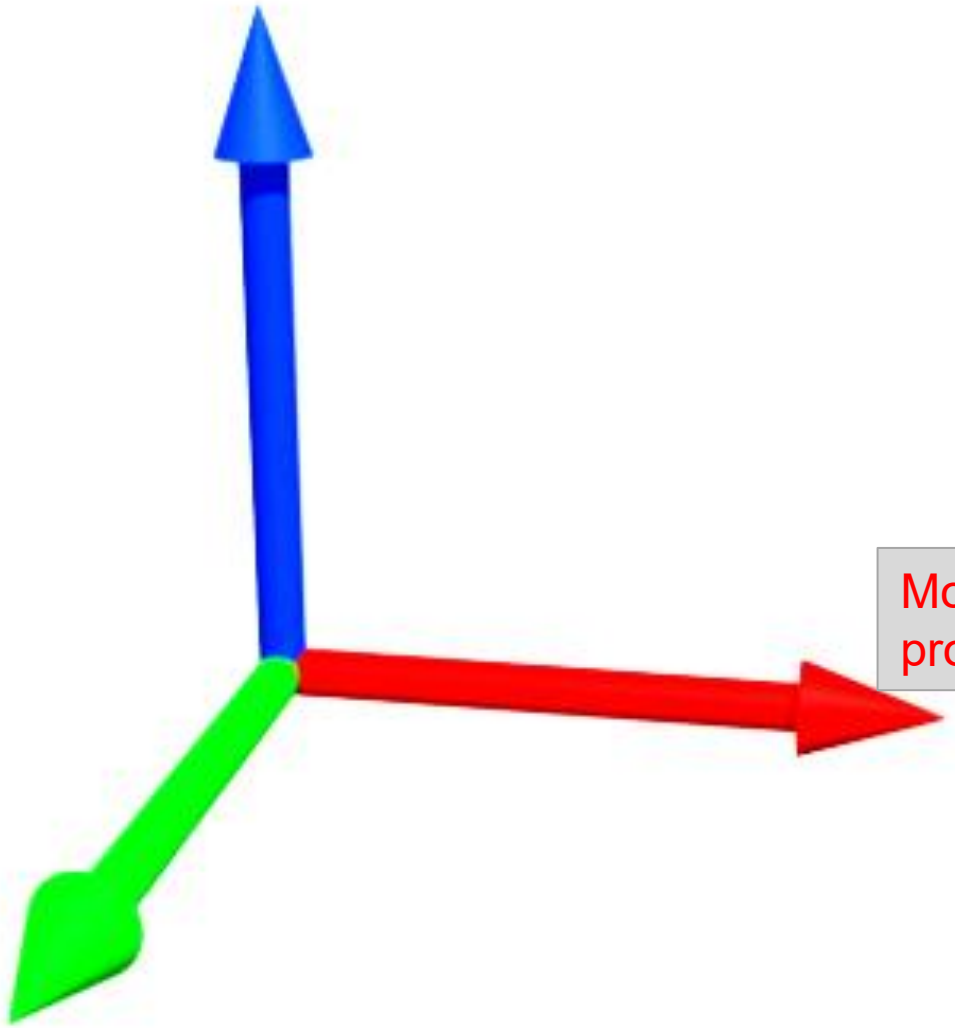
How can these dangers be mitigated?

# Emphasising learning and attainment

- CENOs do not represent a curriculum for teaching
- Rather they represent key foundational ideas that need to be well understood and connected, in order for further progress to be possible
- This makes illustrating depth and breadth in attaining these outcomes important

**Depth**: Outcomes based on solving simple and more complex problems
**Breadth**: Variety of problems that children need to encounter, become familiar with and solve

# Illustrating depth: Simpler/more complex problems



More complex problems

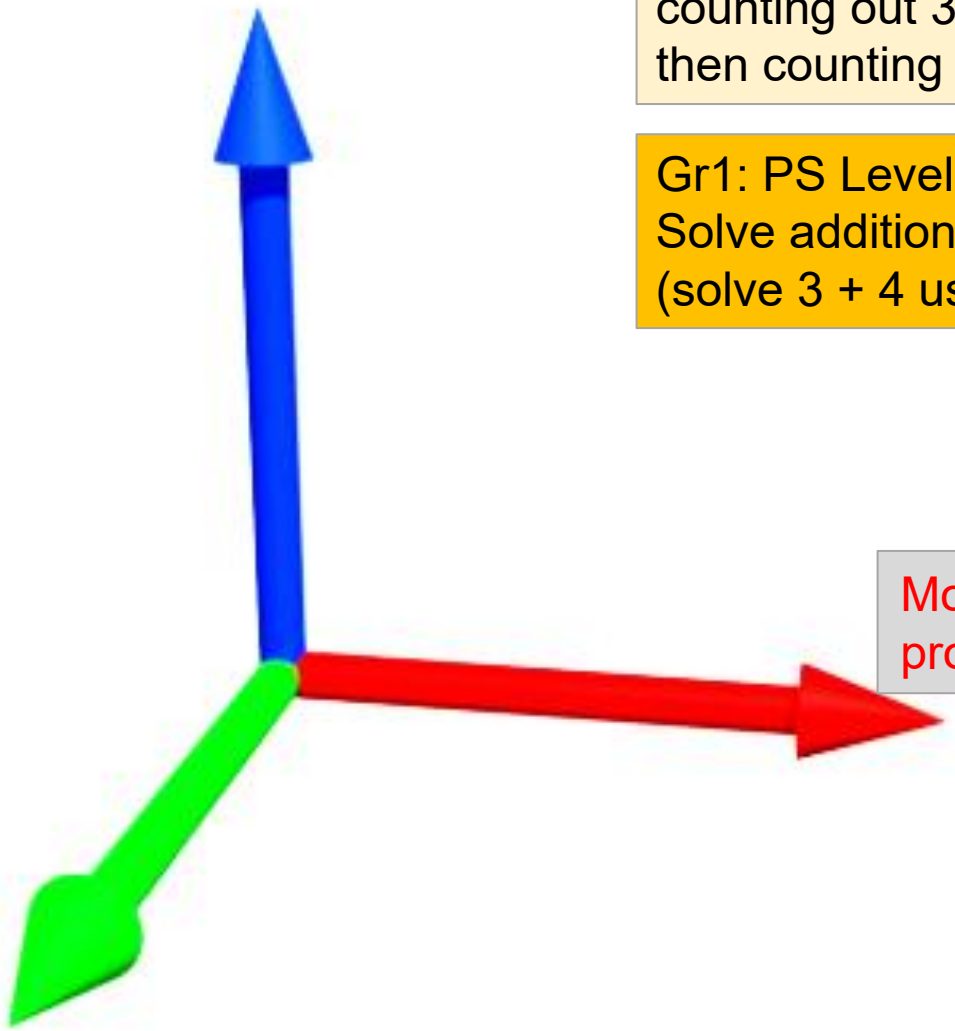
Gr1: PS Level 1
solve problems when the result is unknown (3 + 4 = _)

Gr1: PS Level 2
solve problems when the change/start is unknown (3 + _ = 7)

More efficient strategies

Gr1: PS Level 1
Solve addition/subtraction problems by unit counting (solve 3 + 4 by counting out 3 fingers on one hand, 4 fingers on the other hand, and then counting all fingers to get 7)

Gr1: PS Level 2
Solve addition/subtraction problems using known and derived facts (solve 3 + 4 using 3 + 3 = 6 as a known fact **fluency** and adding 1)



More complex problems

Gr1: PS Level 1
solve problems when the result is unknown (3 + 4 = _)

Gr1: PS Level 2
solve problems when the change/start is unknown (3 + _ = 7)

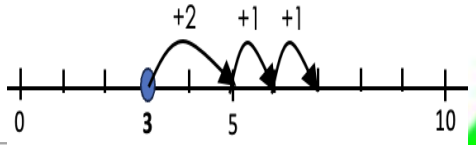
More efficient strategies

Gr1: PS Level 1
Solve addition/subtraction problems by unit counting (solve 3 + 4 by counting out 3 fingers on one hand, 4 fingers on the other hand, and then counting all fingers to get 7)

Gr1: PS Level 2
Solve addition/subtraction problems using known and derived facts (solve 3 + 4 using 3 + 3 = 6 as a known fact **fluency** and adding 1)

Gr1: PS Level 1
Make/ draw/ work with initial, more concrete reps

Gr1: PS Level 2
Make/ draw/ work with more abstract reps




More abstract representations

More complex problems

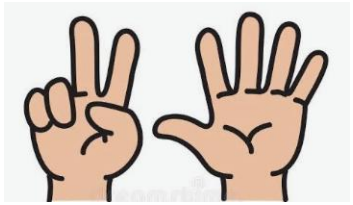
Gr1: PS Level 1
solve problems when the result is unknown (3 + 4 = _)

Gr1: PS Level 2
solve problems when the change/start is unknown (3 + _ = 7)

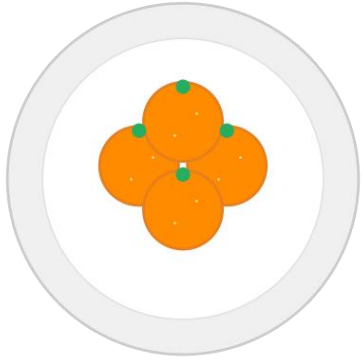
Moves from simpler to more complex language interpretation also incorporated

# Breadth:
# Devising variety of tasks for each fluency and problem-solving outcome

- Add/subtract 1 to/from numbers up to 20



7 fingers open
Open 1 more finger
_ fingers now open



Mia eats 1 orange.
How many oranges left now?

*10 + 1 = _*

*15 - 1 = _*

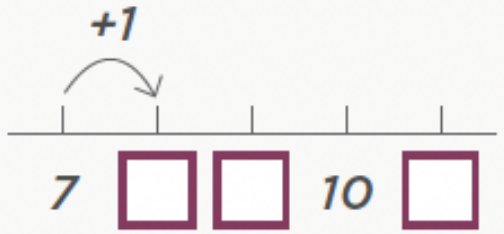
*6 + 1 = _*

*18 - 1 = _*

*One less than 6 is _*

*One more than 19 is _*

*One jump back from 11 is _*




Illustrative items included with CENOs

| | |
|---|---|
| 2 | 1 |
| ? | |

Incorporating spatial representations

# Emphasis on connecting ideas

To avoid focus on discrete outcomes for attainment, emphasis is placed on a small number of features that are predicated on building connections across outcomes:

- Representations (p.37)
- Place value (p.41)
- Efficient calculation (p.46)

Inclusion of illustrative tasks set in the context of Shape and Space and Data Handling.

Goal to support teachers with moves between simpler and more complex task formulations and recognize and support progression.

# NRD programme and CENOs

NRD fund: Seeks to improve foundational numeracy outcomes in sub-Saharan Africa.

Implementing partners in:

- Ethiopia
- Ghana
- Malawi
- South Africa
- Tanzania
- Uganda
- Zambia

CENOs being used to develop teaching materials and assessment tasks, and to assess progress of foundational numeracy

# Thank you!

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