

Data en statistische diagrammen in het po

Stippendiagram, casus-staafdiagram

Oogbewegingen van leerlingen

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en Analytisch Vermogen van Professionals,
Lonneke.boels@hu.nl

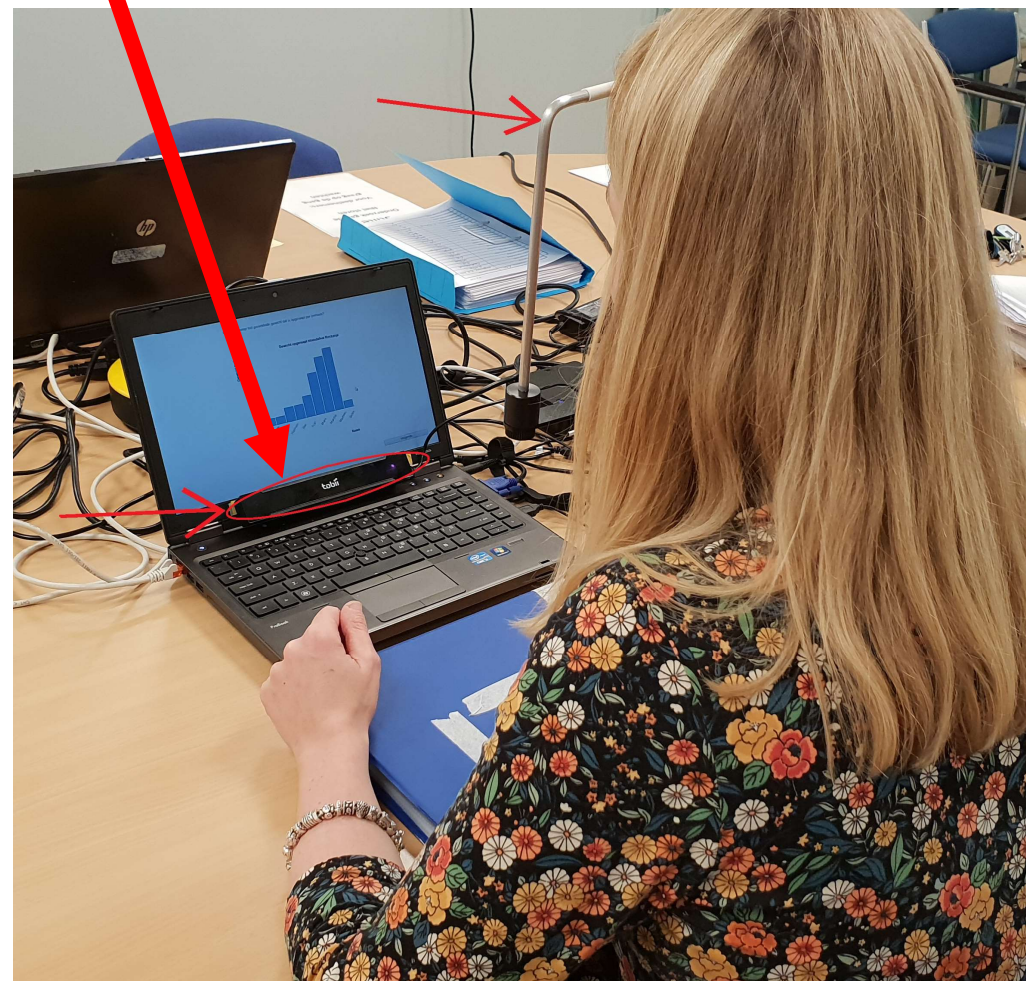


Utrecht University

Dynamics of Youth



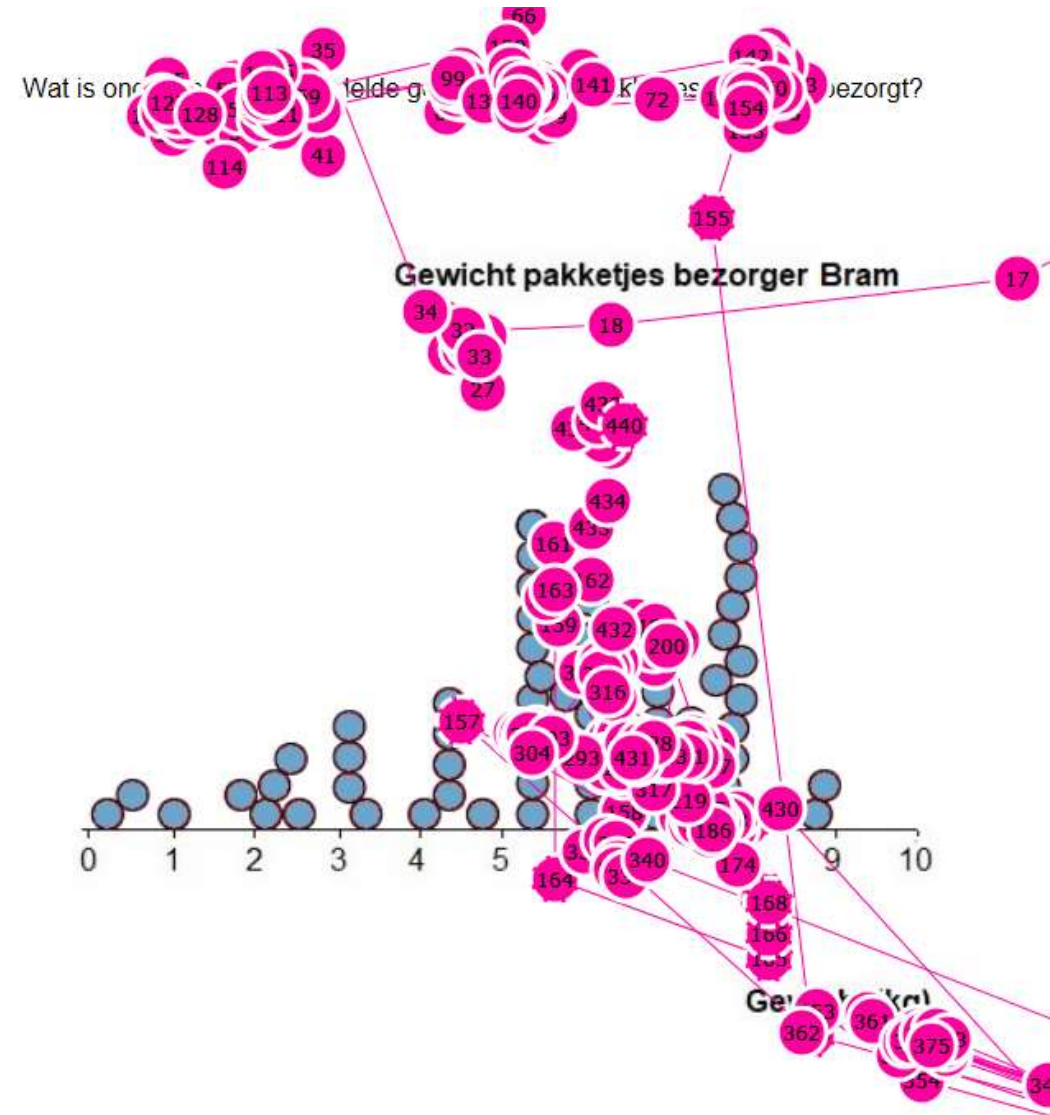
Science



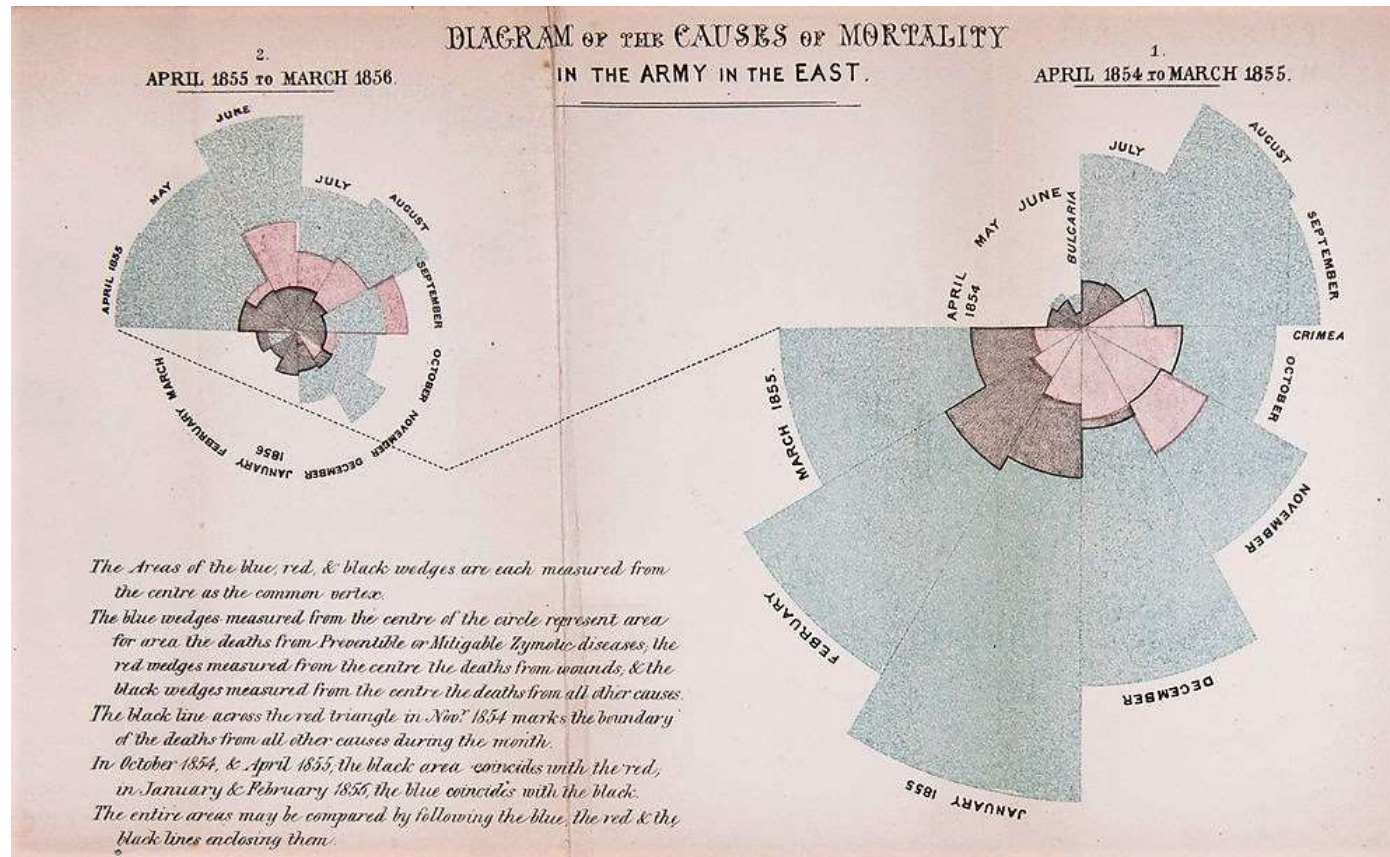
Overzicht presentatie

- Florence Nightingale
- Doorlopende leerlijn po–vo
- Oogbewegingen leerlingen en taken
- Lesmateriaal po en pabo
- Reclame – afronding

8 juni 2024



Een goede grafiek kan levens redden

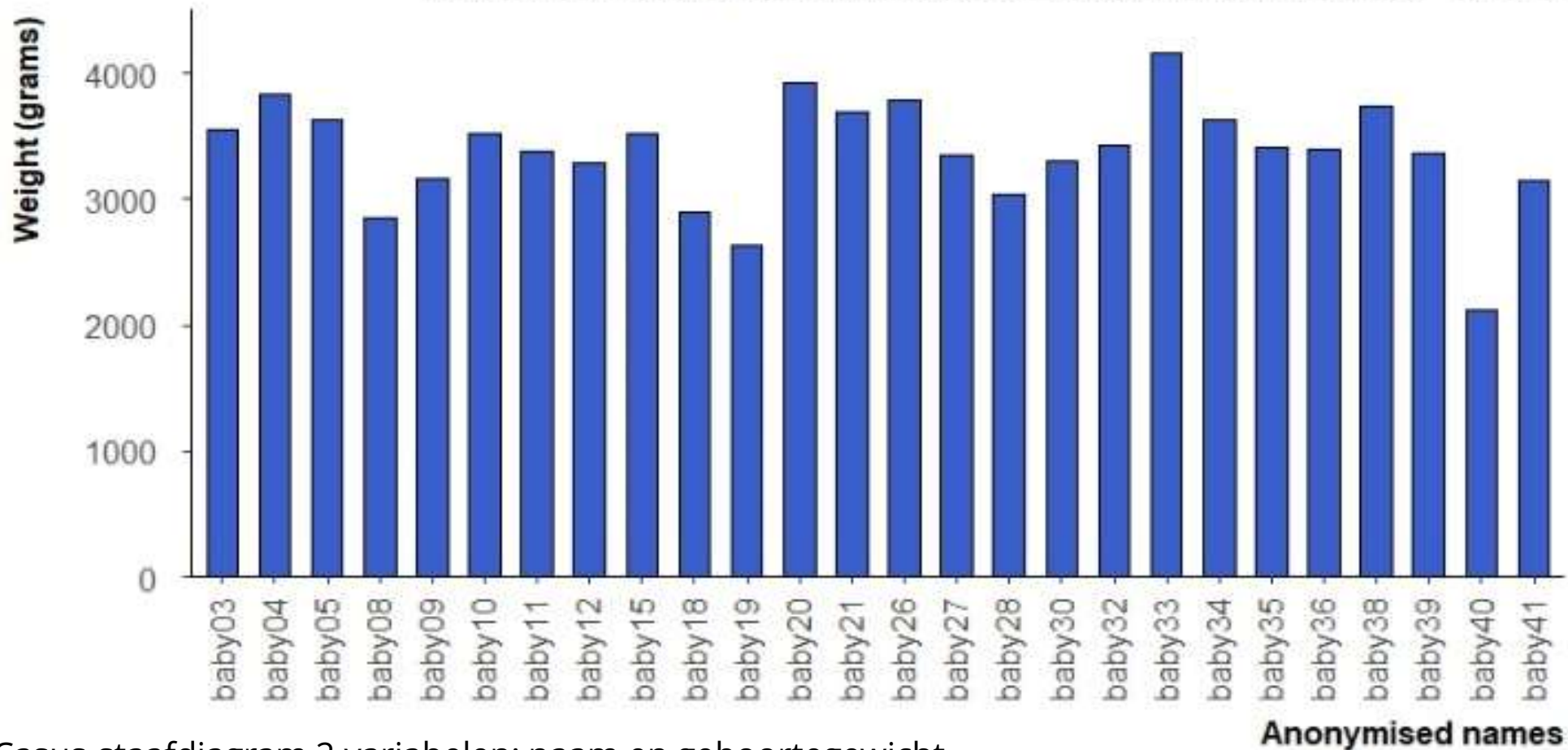


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Leerlijn bovenbouw PO – onderbouw VO

Groep 6–8

Birth weight baby girls, hospital, Queensland, Australia, December 18, 1997

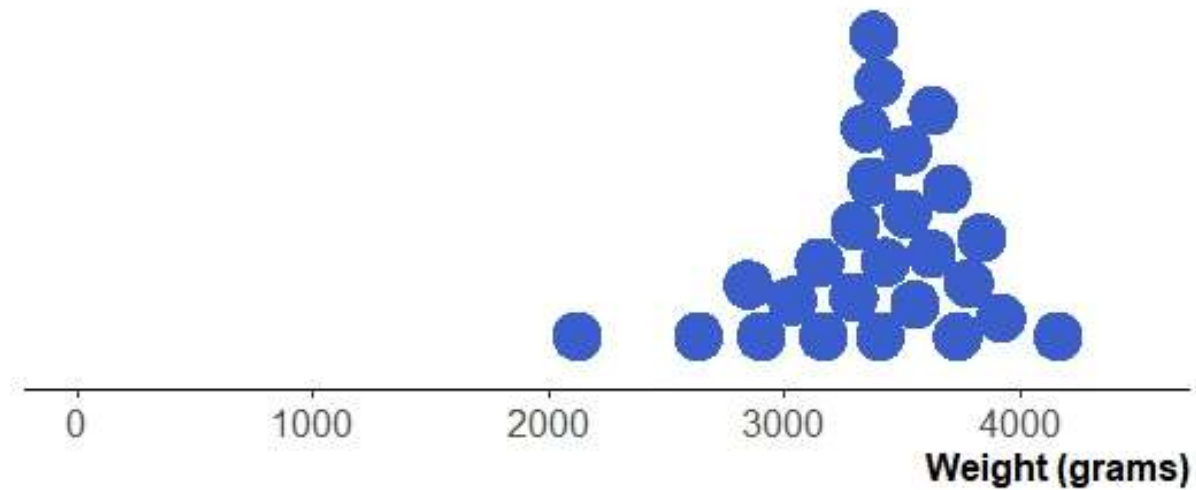


Casus-staafdiagram 2 variabelen: naam en geboortegewicht

(Bron: Boels, 2023; Boels et al., 2019).

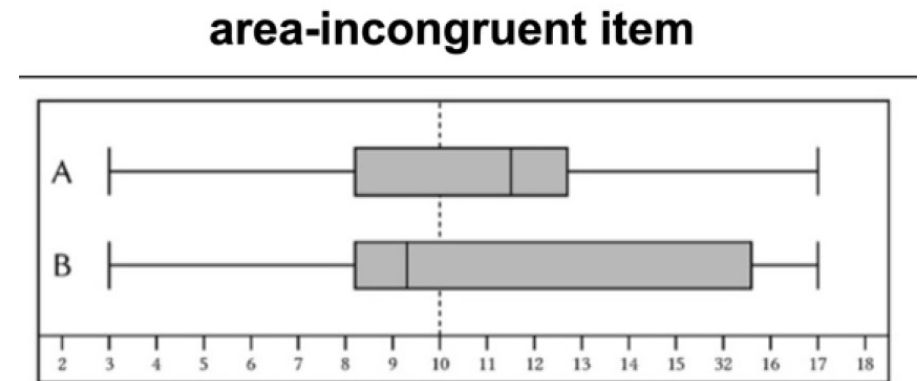
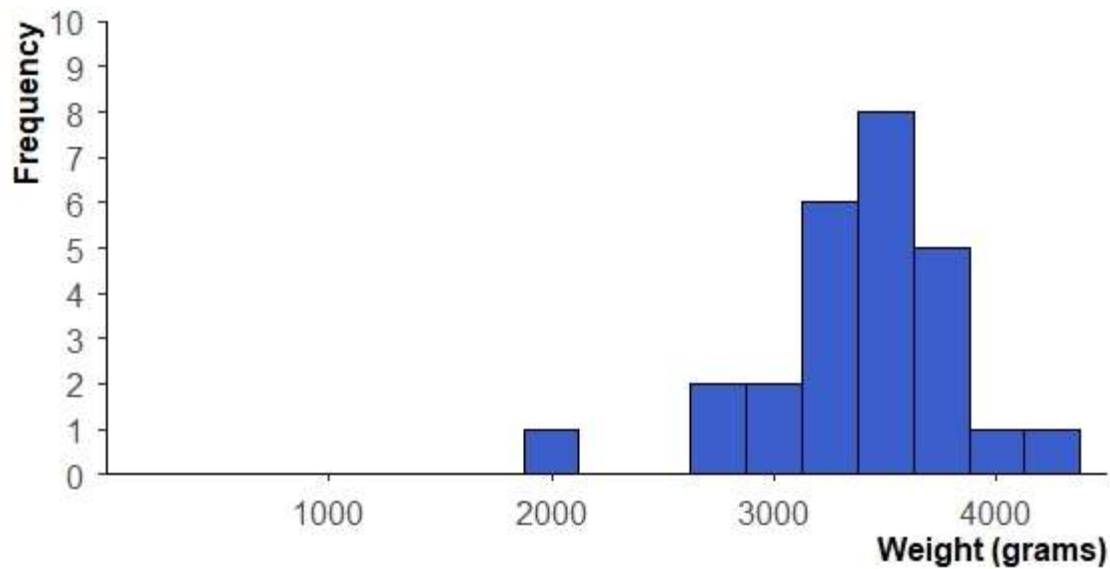
(Geboortegewicht data uit Dunn, 2018).

Groep 6–8; onderbouw vo



(Geboortegewicht data uit Dunn, 2018).

3^e – 4 klas vo



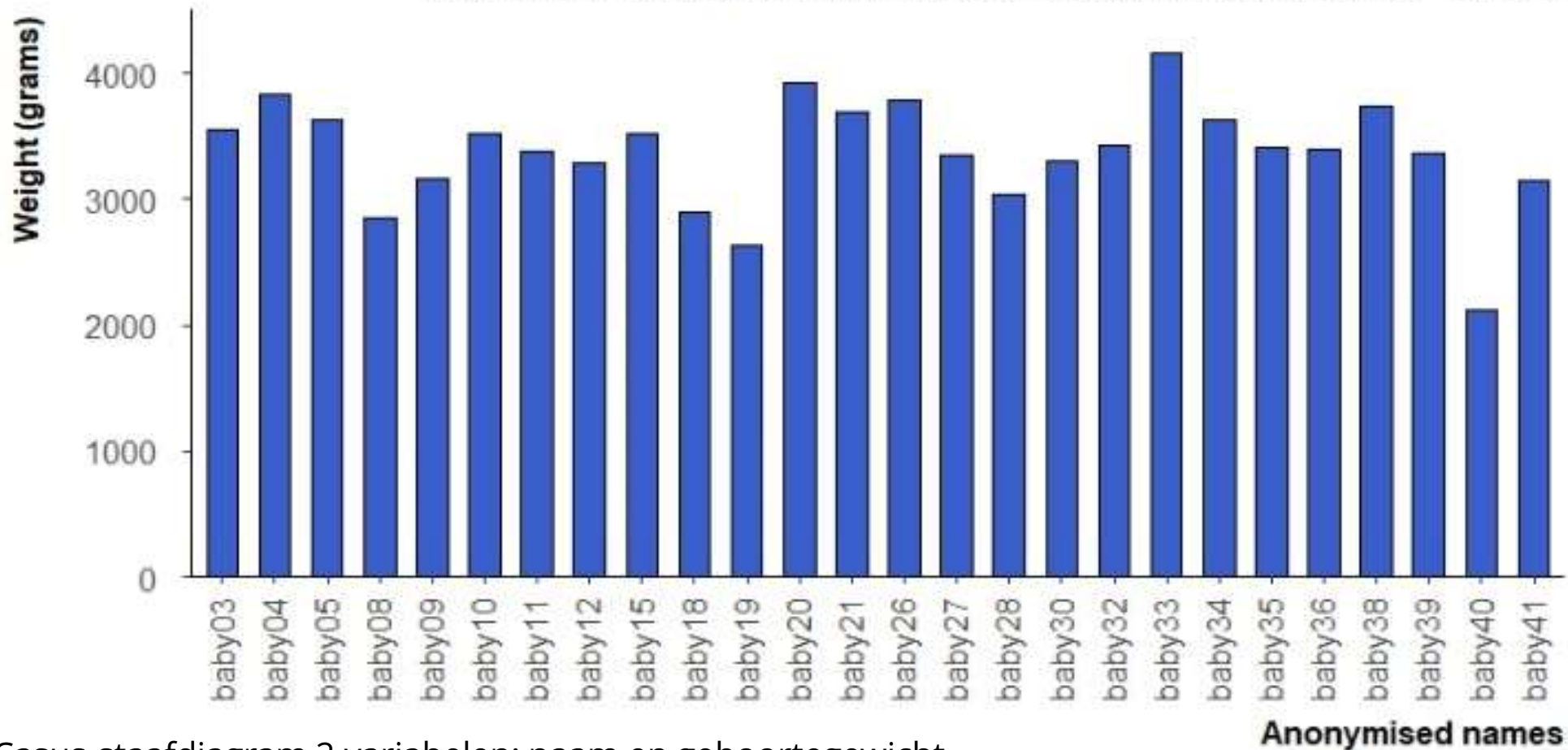
(Geboortegewicht data uit Dunn, 2018). Histogram (links) en boxplot (rechts)
Boxplots uit Abt et al. (2023).

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Oogbewegingen leerlingen
Casus-staafdiagram

Groep 6–8

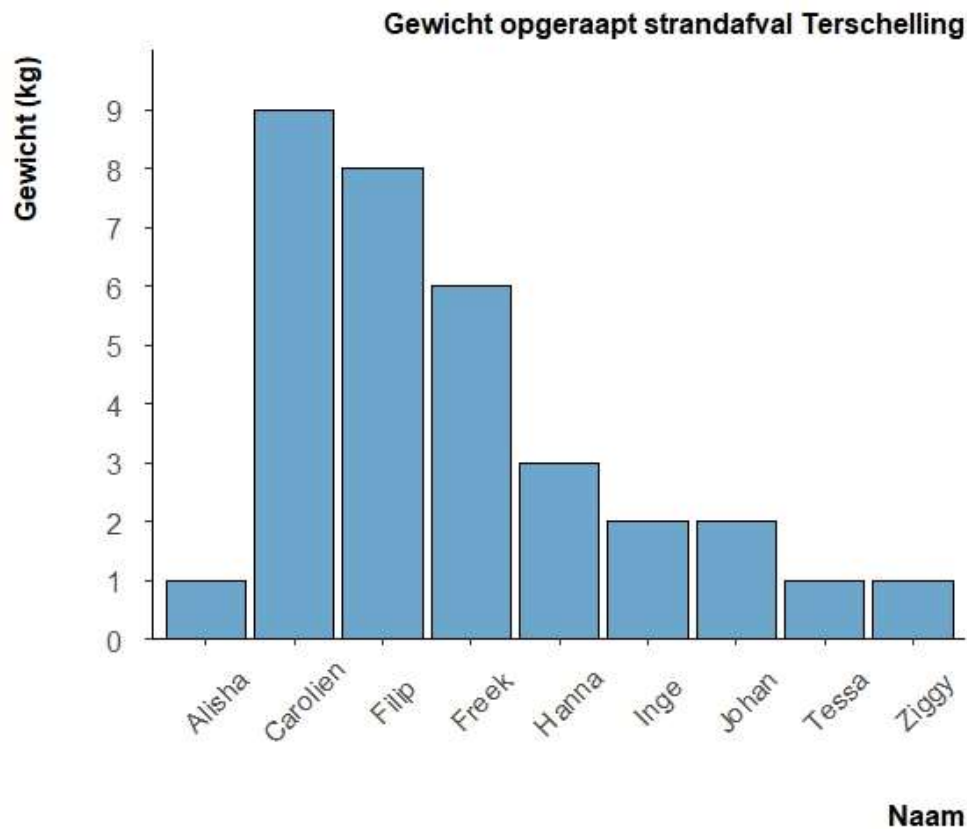
Birth weight baby girls, hospital, Queensland, Australia, December 18, 1997



Casus-staafdiagram 2 variabelen: naam en geboortegewicht

(Geboortegewicht data uit Dunn, 2018).

Casus-staafdiagram



Wat is ongeveer het gemiddelde gewicht dat is opgeraapt per persoon?

Boels (2023). Histograms. An educational eye.

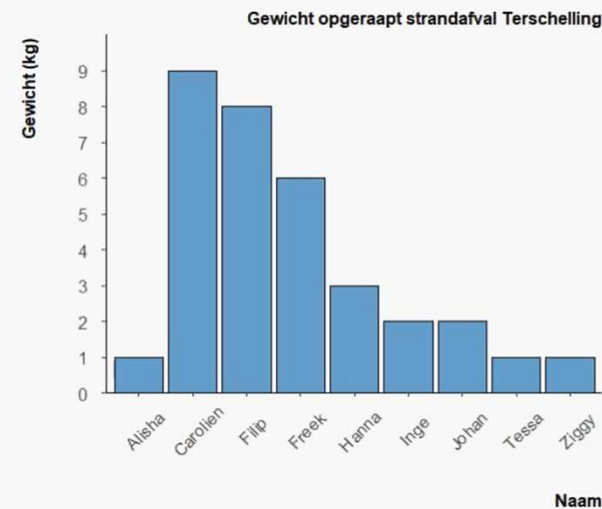
Casus-staafdiagram 2 variabelen: naam en gewicht opgeraapt strandafval

Data

Video van de oogbewegingen van leerling L03 (ca. 15 seconden)

Achteraf:
L03: [...] kijken hoe hoog de tabellen [staven] in de grafiek zijn. En dan weer een beetje in het midden kiezen want er zijn, er zijn vier lagen en vier een beetje hoger. Dus dan is het gemiddelde logisch [?] als het in het midden van de hoogste tabel [staaf] is.
Onderzoeker: Ja, ok. En zo kwam je aan het antwoord, wat was het ook alweer, vijf.
L03: Ja.

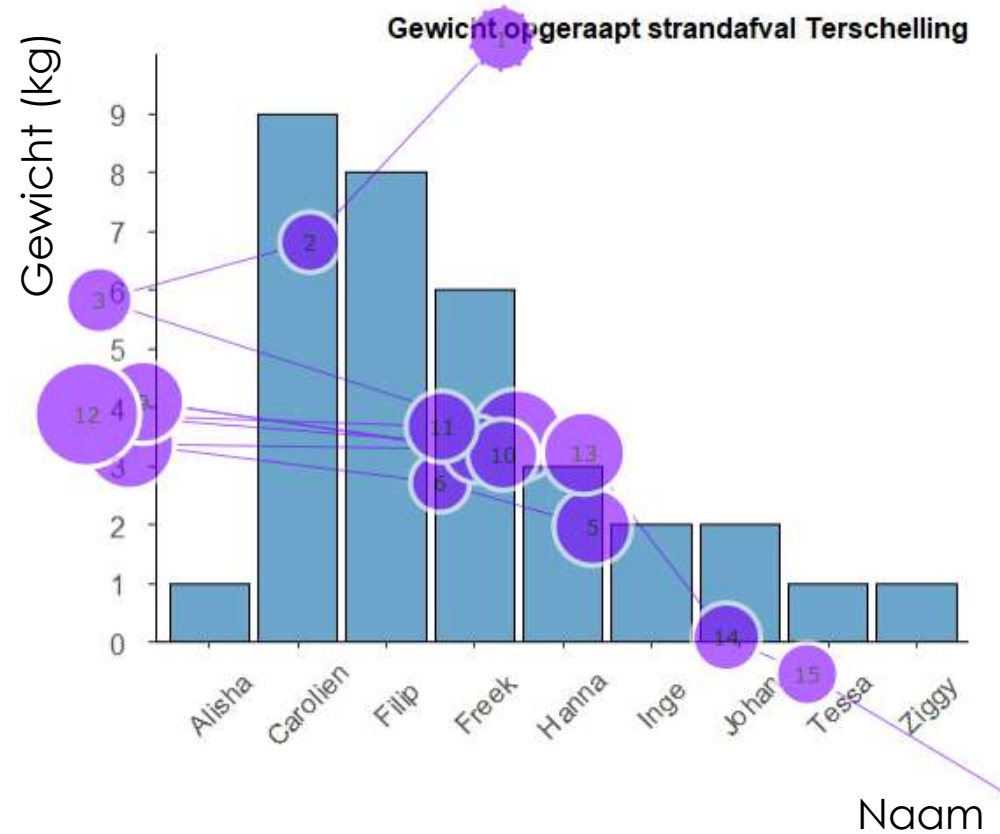
Wat is ongeveer het gemiddelde gewicht dat is opgeraapt per persoon?



Correcte strategie - Item 04

Boels (2023). Histograms. An educational eye.

Wat is ongeveer het gemiddelde gewicht dat is opgeraapt per persoon?

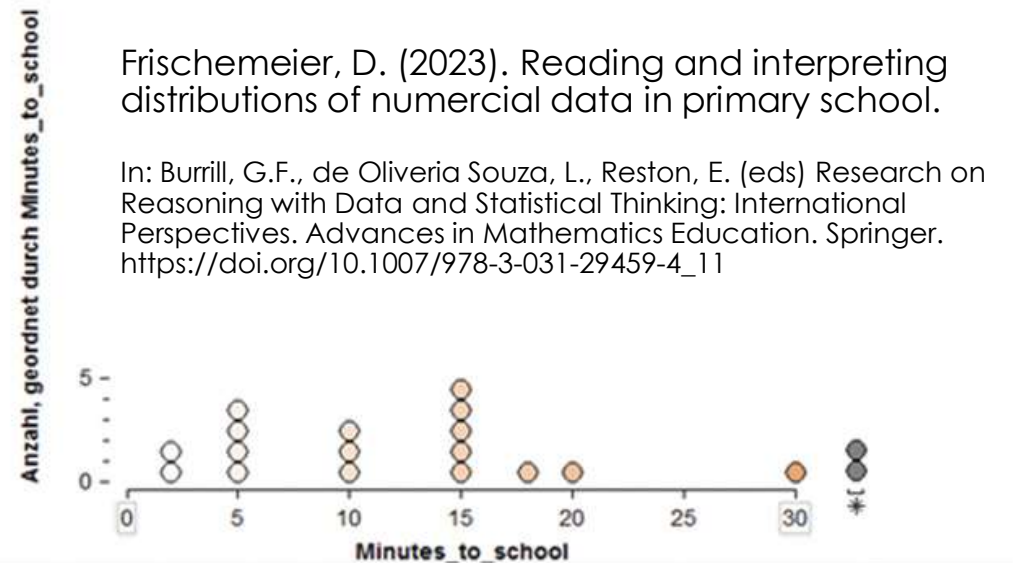
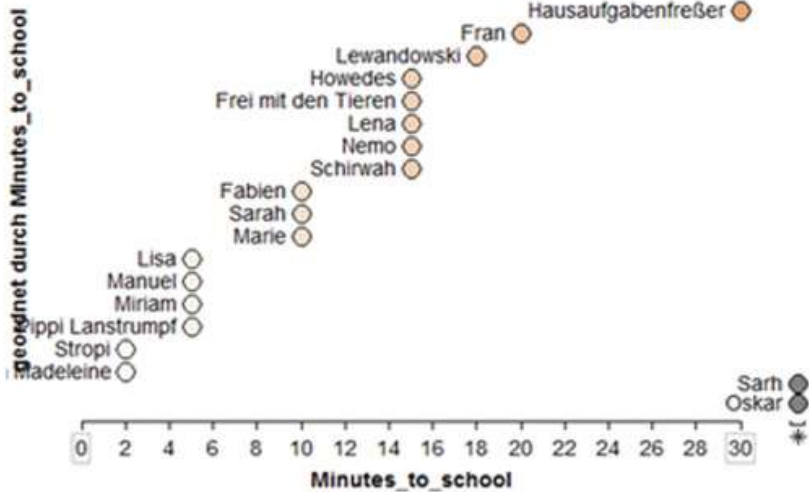
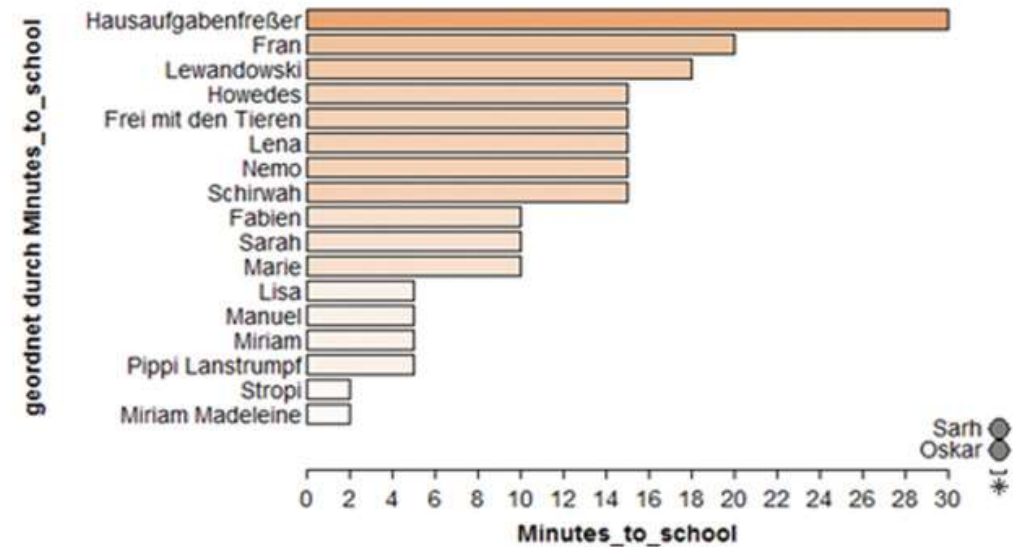
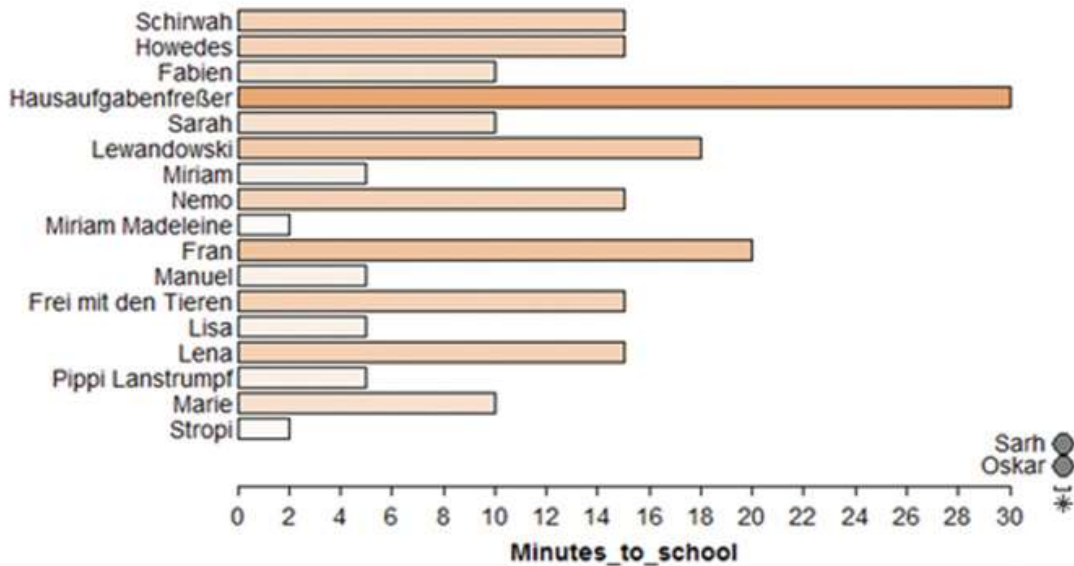


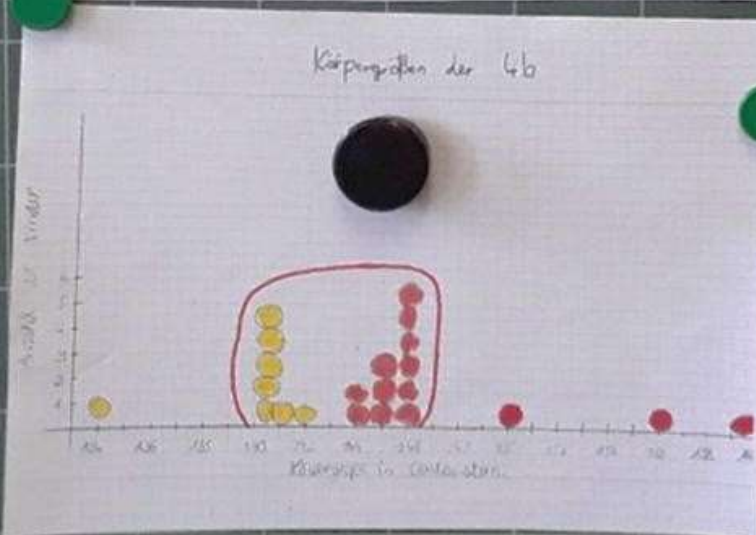
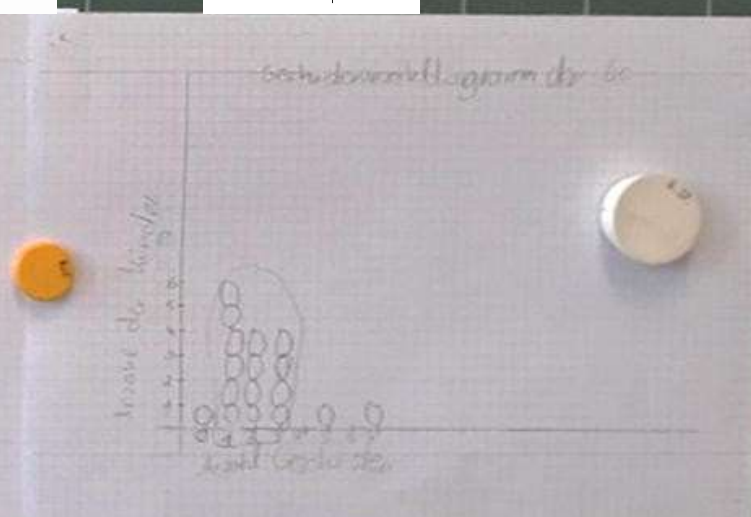
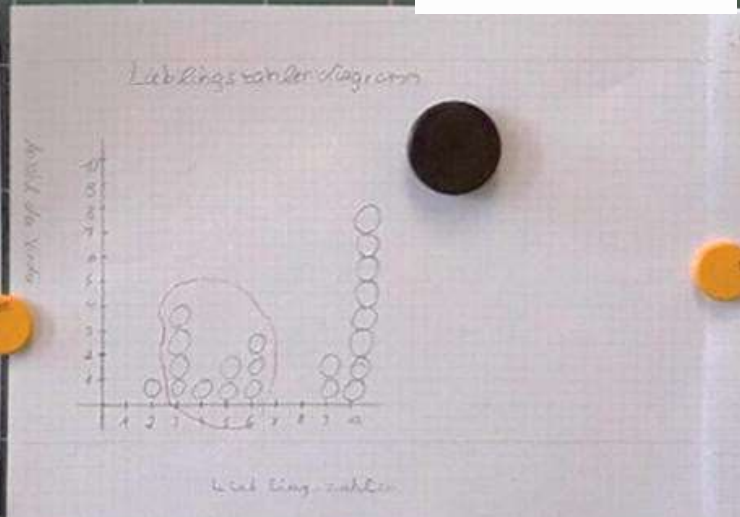
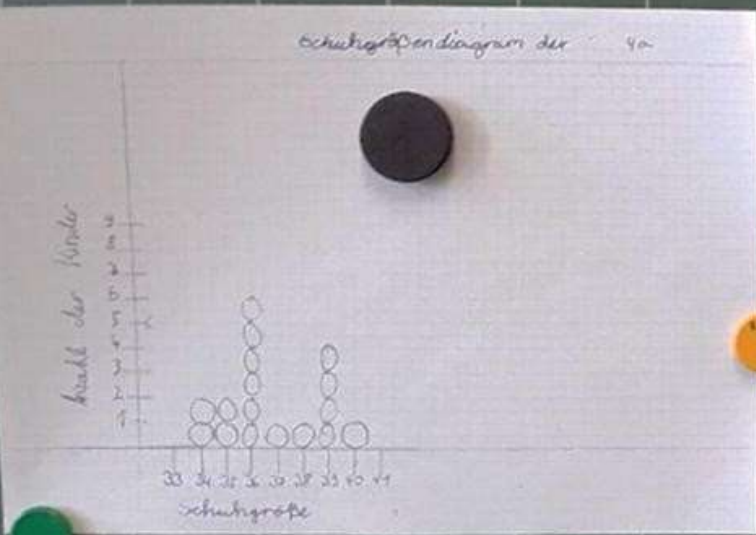
Oogbewegingspatronen
Gaze plot – L03 – Item 4

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Taken leerlingen

Casus-staafdiagram naar stippendiagram





Frischemeier, D. (2023).

Groep 7

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 Miele Complete C3 Silence... €299.00 Coolblue Free shipping ★★★★★ (187) By Producthero	 SALE BISSELL CrossWave C3... €238.99 €339 Bissell Free shipping By Producthero	 SALE Dyson V8 Origin Stofzuiger €279.00 €329 Dyson Nederland Free shipping Free gift By Google	 Samsung Jet 70 Turbo stofzuiger €289.00 Coolblue Free shipping ★★★★★ (1k+) By Producthero	 Kärcher stofzuiger T 10/... €159.97 Viking Direct +€5.98 shipping ★★★★★ (169) By Productcaster	 Miele Complete C3 PowerLine... €299.00 Coolblue Free shipping ★★★★★ (2k+) By Producthero	 Philips Stofzuiger zonder zak... €175.99 Philips NL DA Free shipping ★★★★★ (816) By Bigshopper	 Miele Complete C3 Cat & Dog... €354.00 Coolblue Free shipping ★★★★★ (33) By Producthero
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 coolblue.nl
https://www.coolblue.nl/stofzuigers

Beste Stofzuigers Juni 2023 - Coolblue: We helpen je kiezen

Bekijk onze Stofzuigers online, of kom langs in een van onze winkels. Ook op zondag en in...

Electronics Retail and Repair Shop - Den Haag - Open - Closes 8 pm



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 Consumentenbond
https://www.consumentenbond.nl

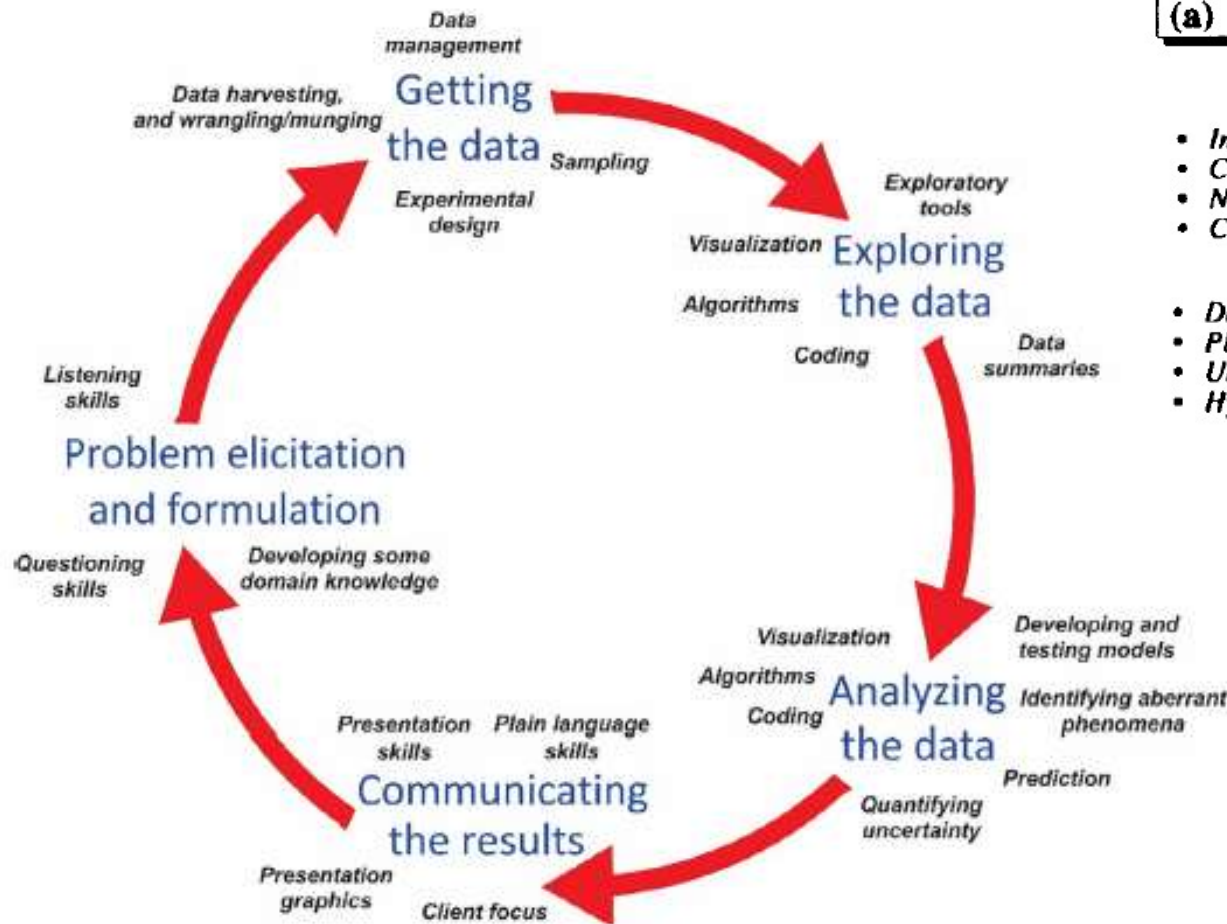
Eerlijk is Eerlijk - Beste stofzuiger 2023 - Voorkom een miskoop

We hebben stofzuigers getest o.a. op gebruiksgemak, geluid, energiegebruik en levensduur.

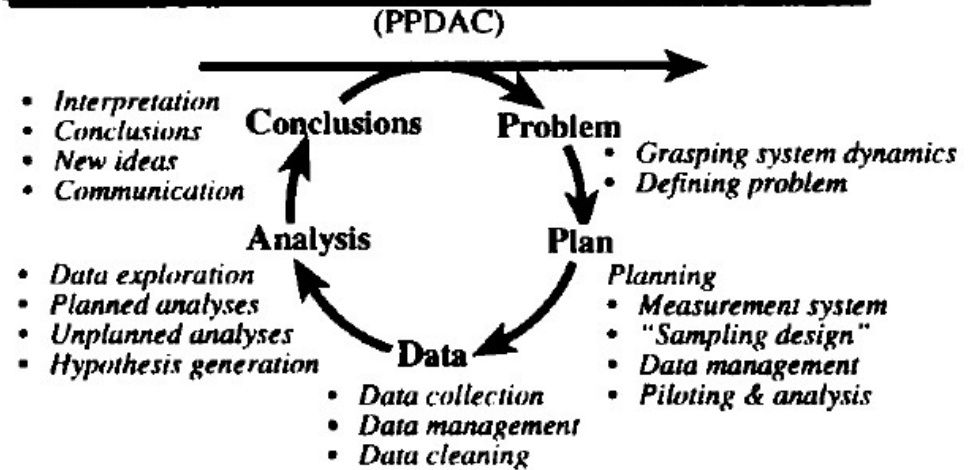
 Cleanipedia.com
https://www.cleanipedia.com/wat4... Translate this page

Wat is de beste stofzuiger voor jou (en je gezin)?





(a) DIMENSION 1: THE INVESTIGATIVE CYCLE



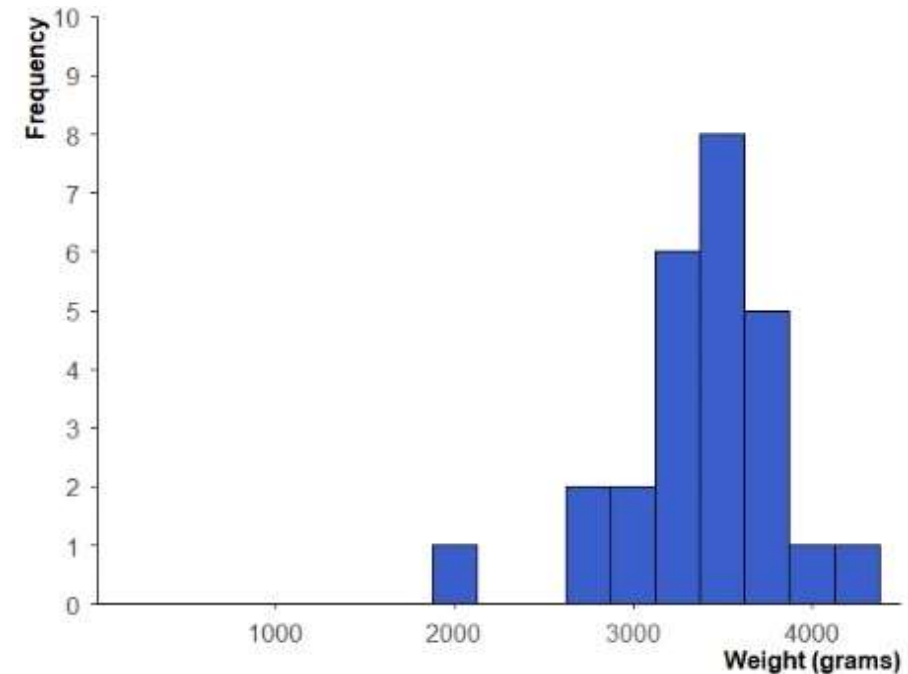
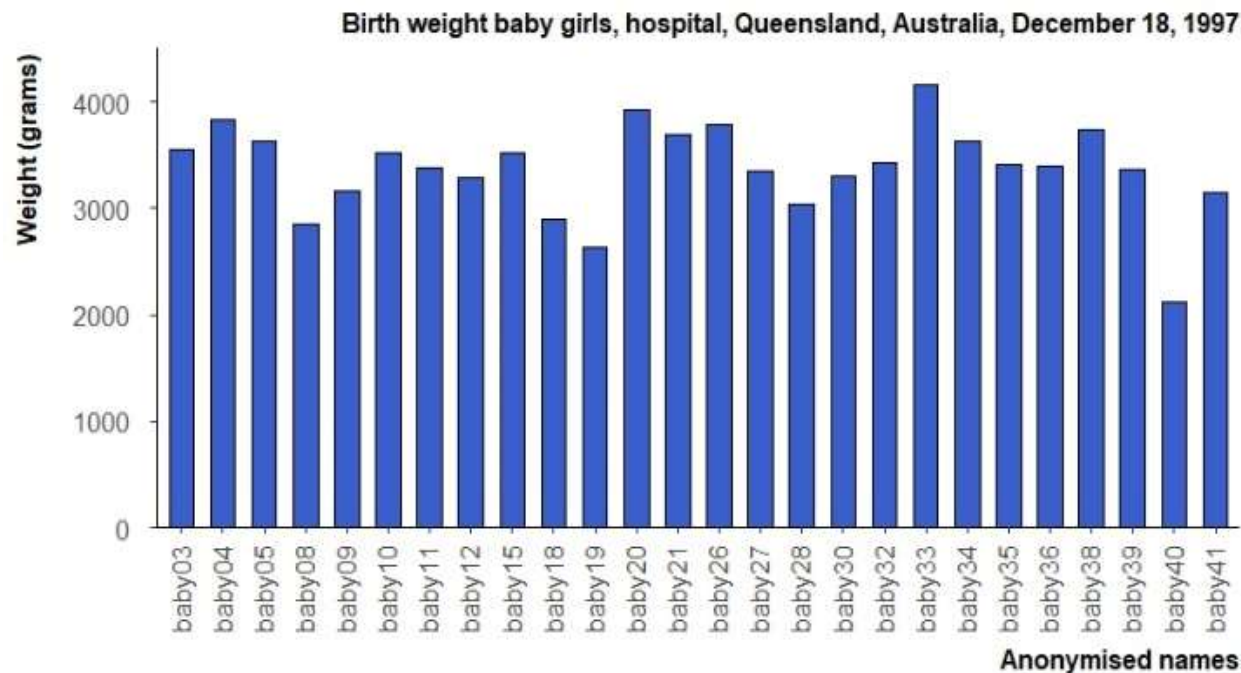
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Intermezzo (roteren en spiegelen)
Van casus-staafdiagram naar
stippendiagram

From a case-value plot to a histogram

a) Case-value plot with two variables: name and birth weight

Histogram



(Birth weight data taken from Dunn, 2018).

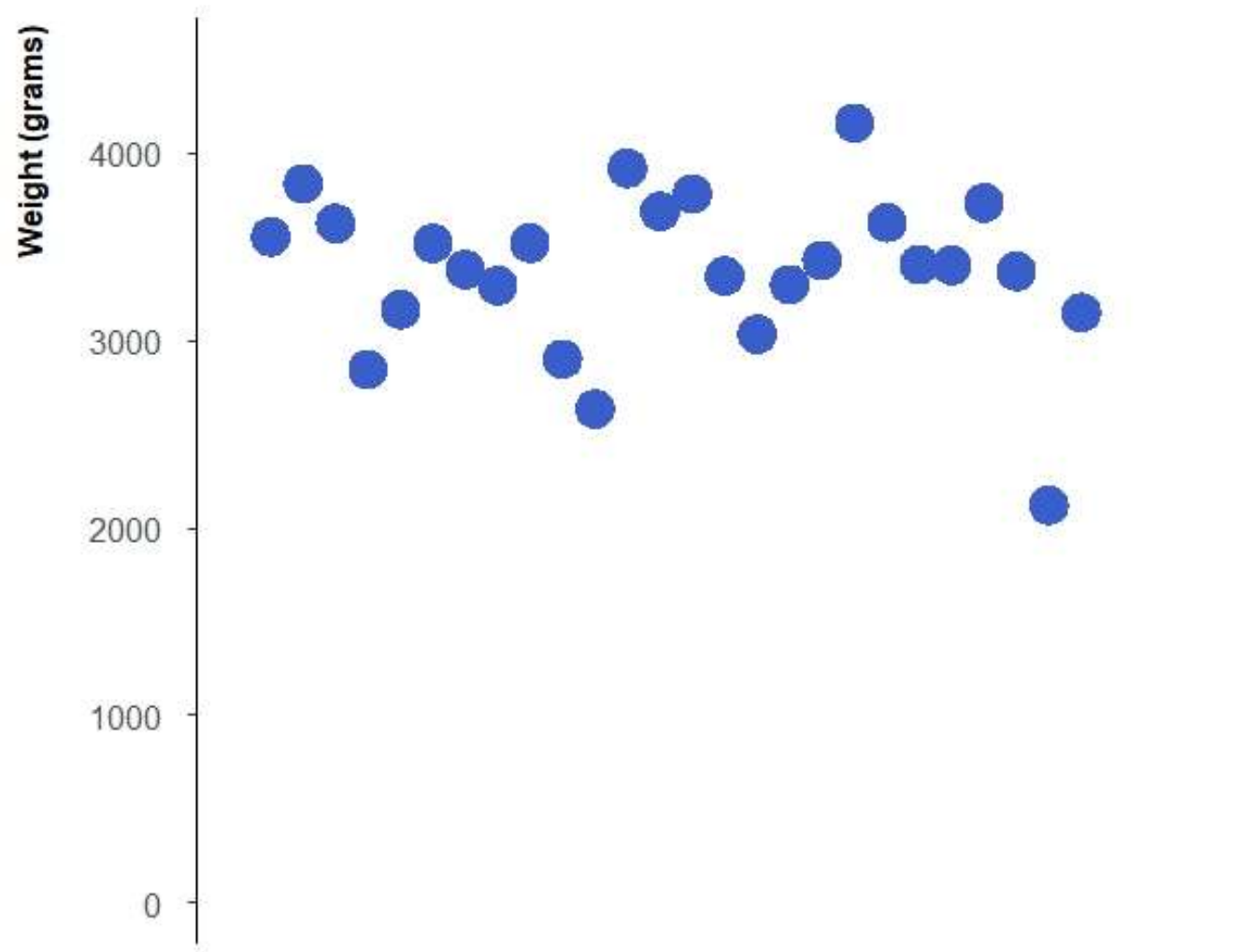
[Boels](#), L., Bakker, A., Van Dooren, W., & Drijvers, P. (2019).

Conceptual difficulties when interpreting histograms: A review.

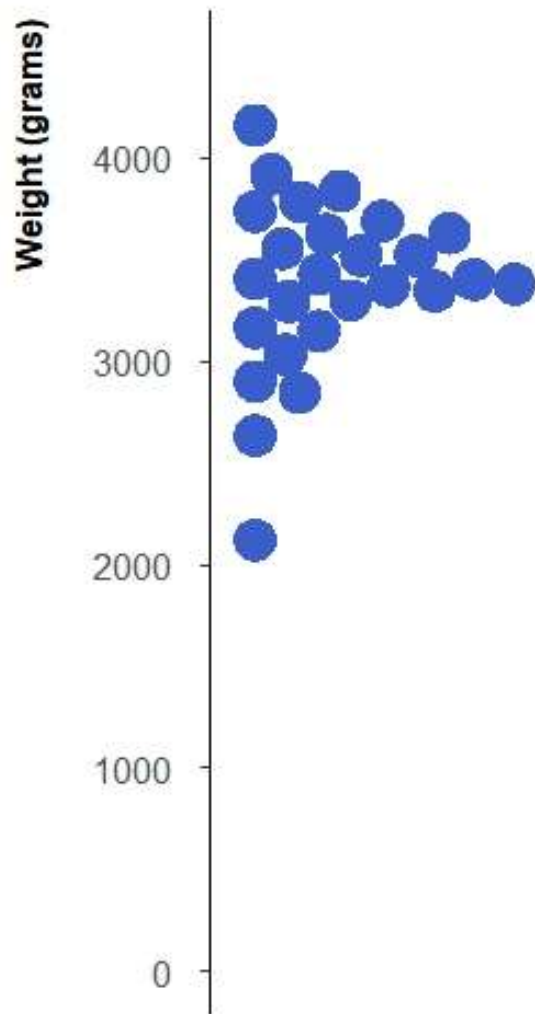
Educational Research Review, 28, Article 100291, 26 p.

<https://doi.org/10.1016/j.edurev.2019.100291>

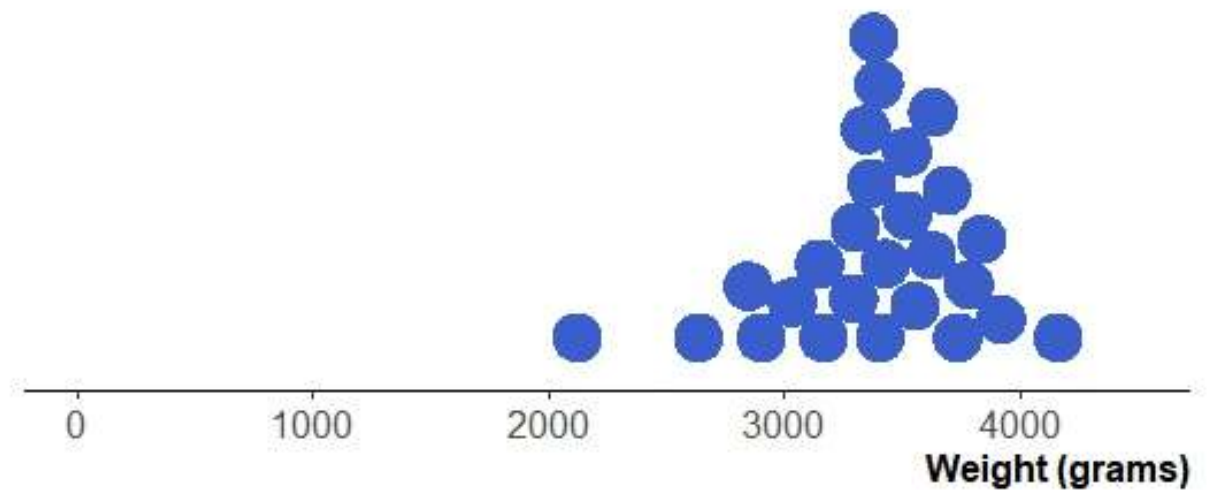
b) univariate graph after removing the variable 'name'



c) all dots are slid to the left into a horizontal dotplot. Note the disappearance of the horizontal axis

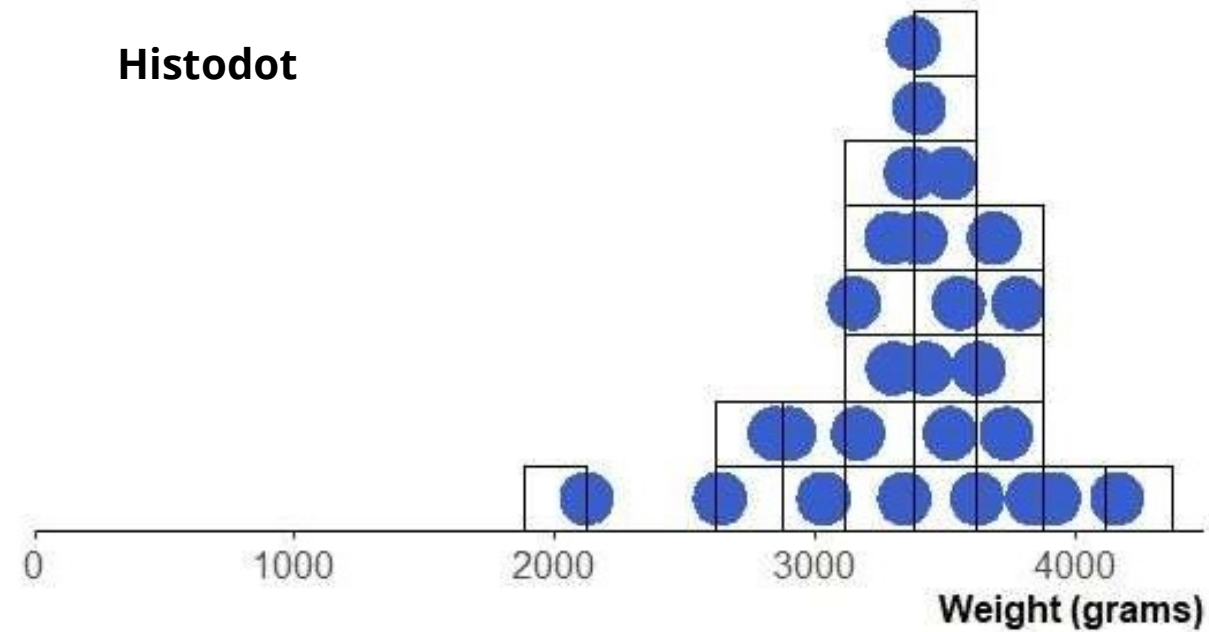


d) messy dotplot after rotating and mirroring of the vertical axis. Note the absence of the vertical axis

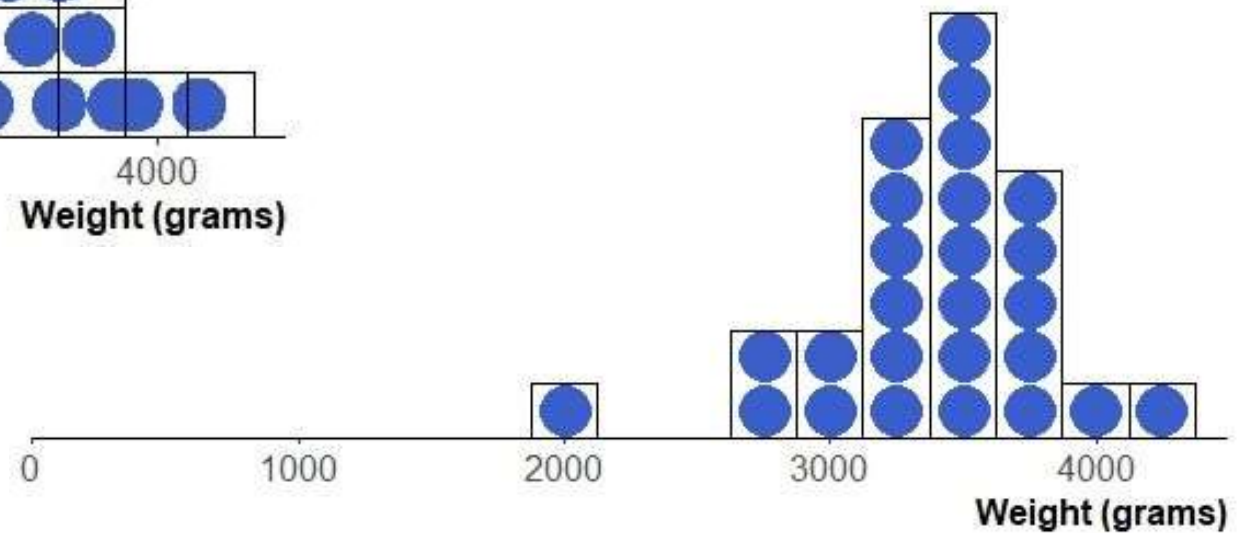


e) binning the dots in a bar; roughly positioned at each measured value

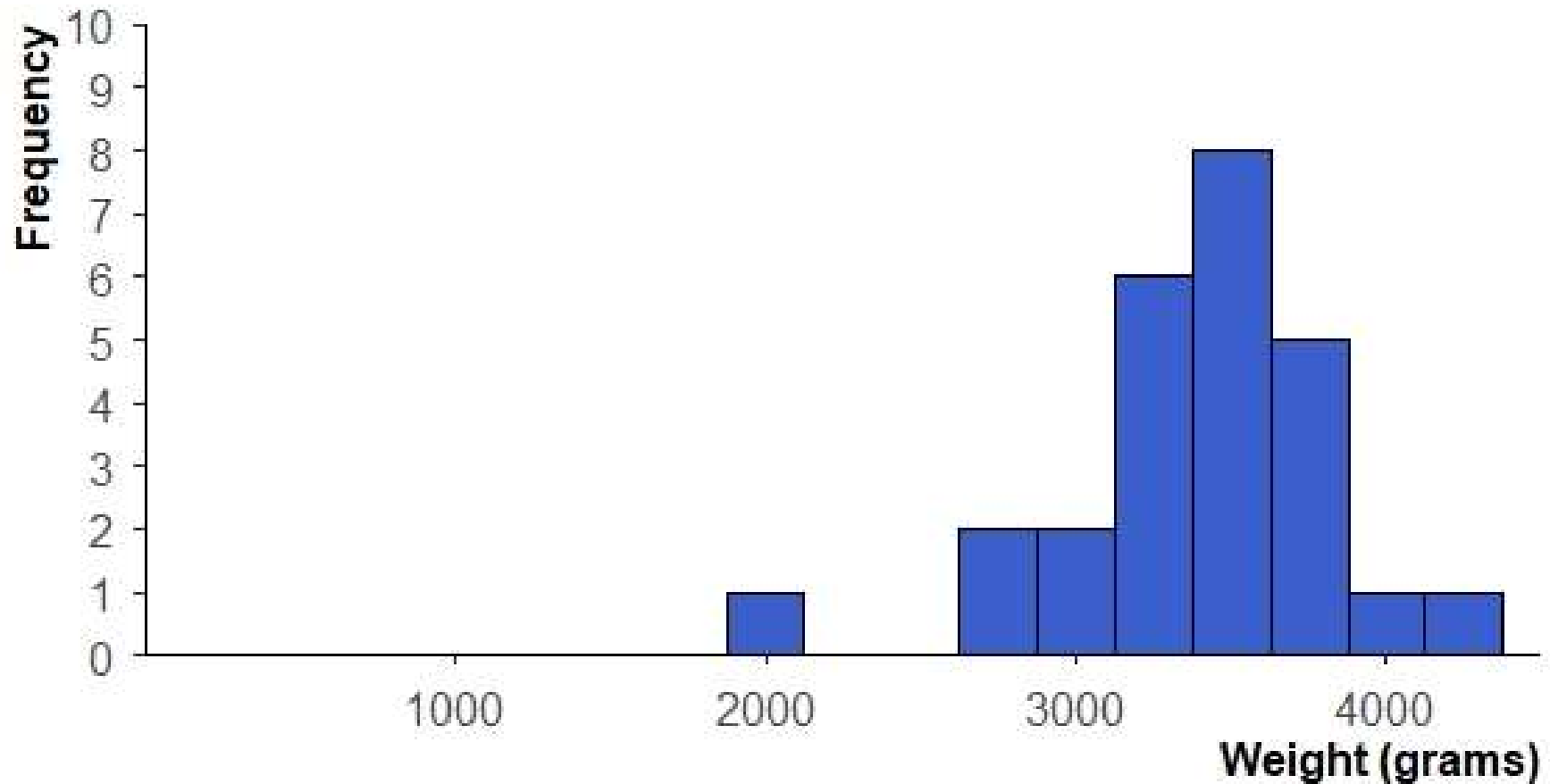
Histodot

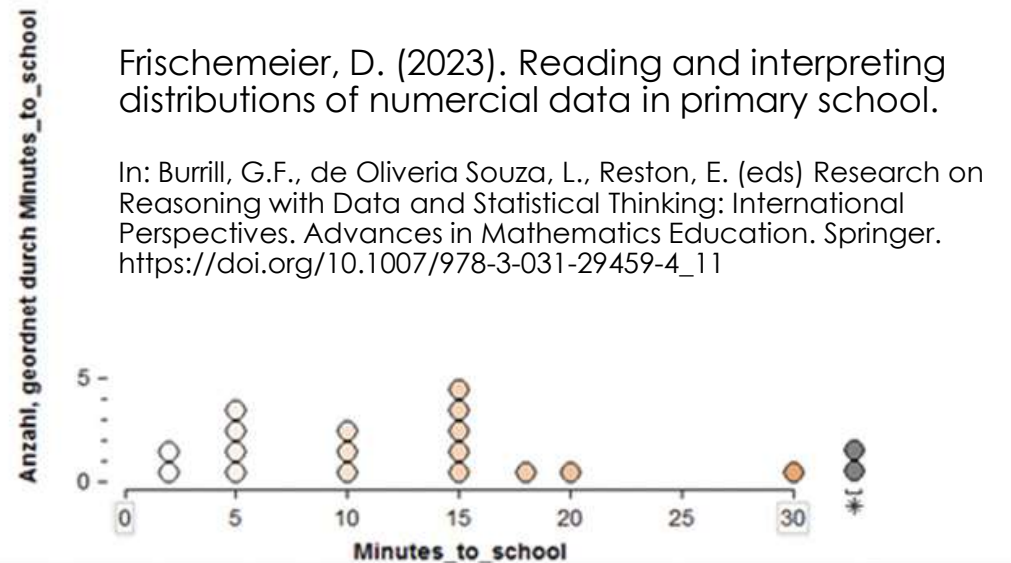
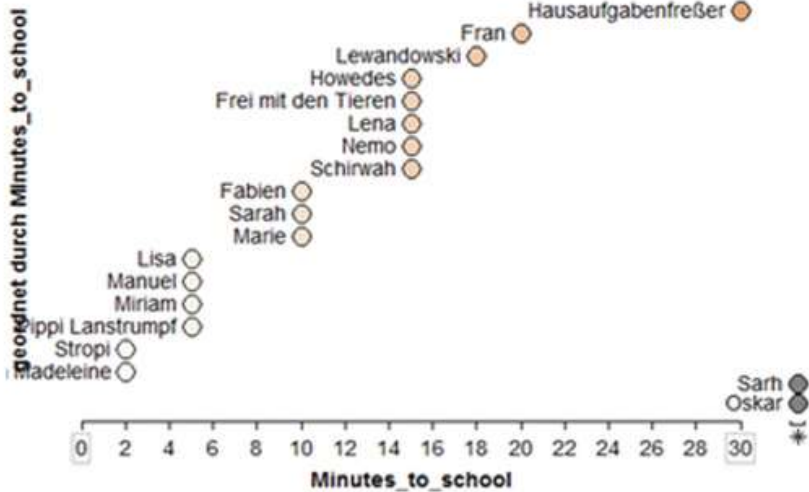
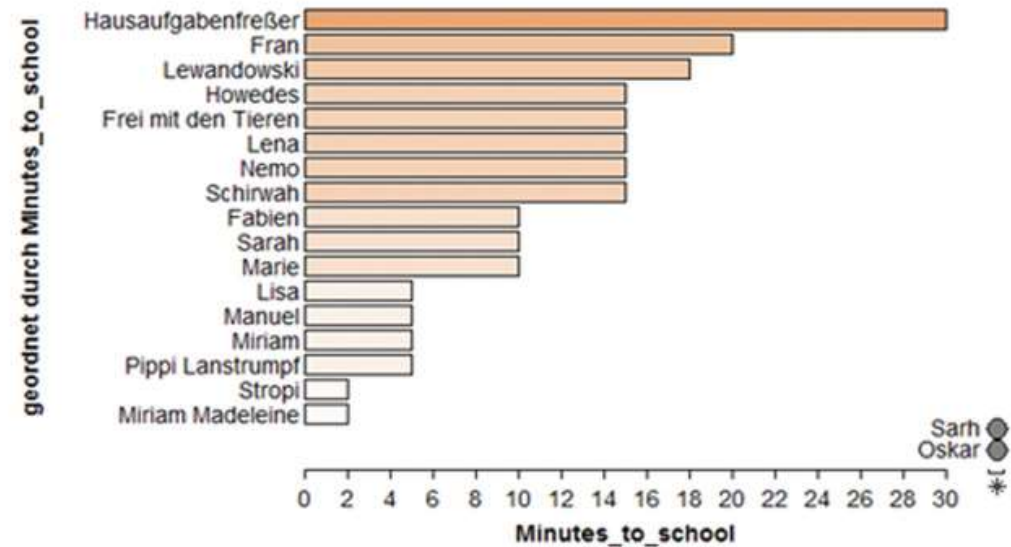
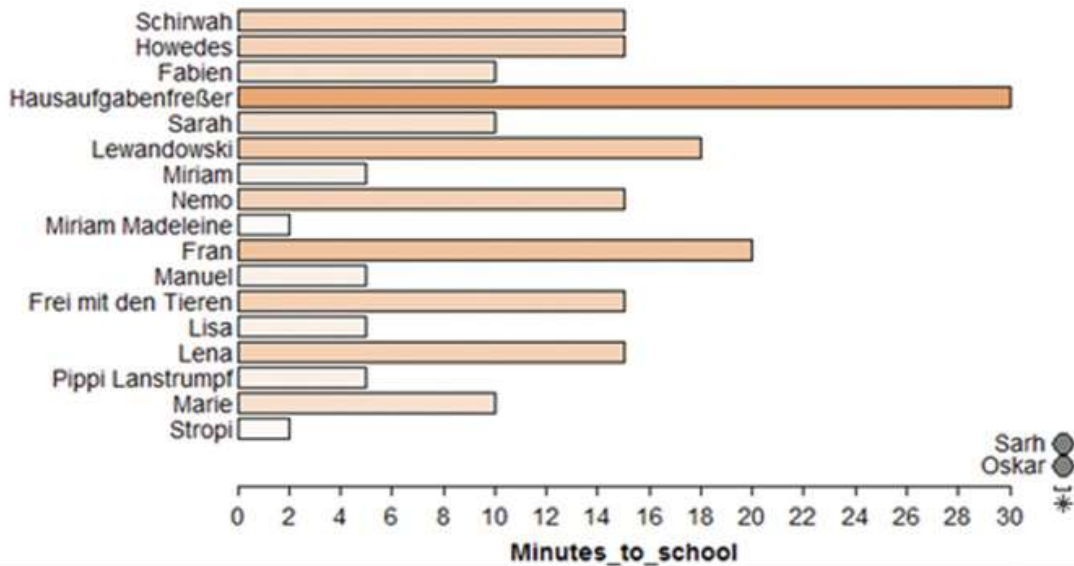


f) grouping the dots in each bar in a stacked dotplot with histogram overlay



g) leaving out the dots, resulting in a histogram. Note the reappearance of a vertical axis in this univariate graph

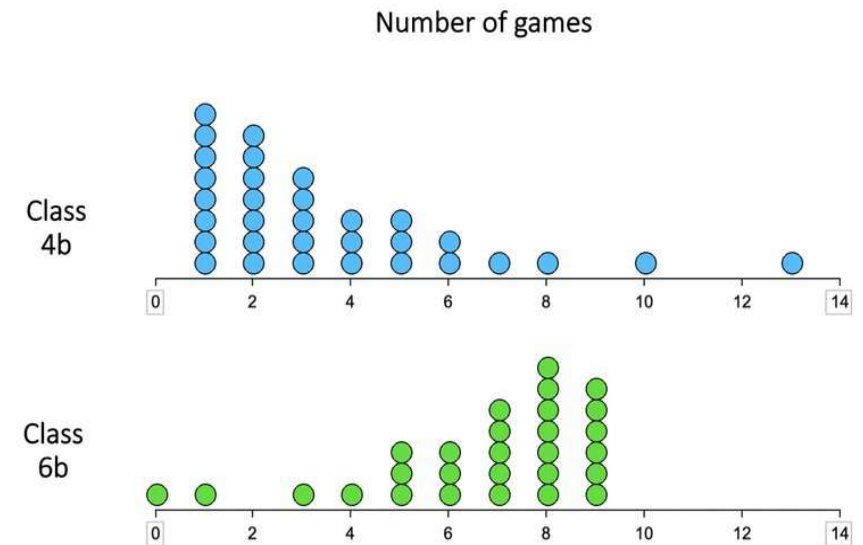
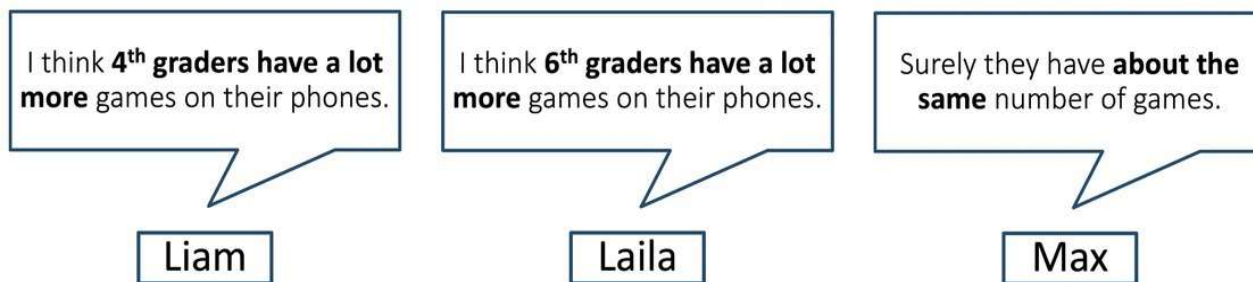




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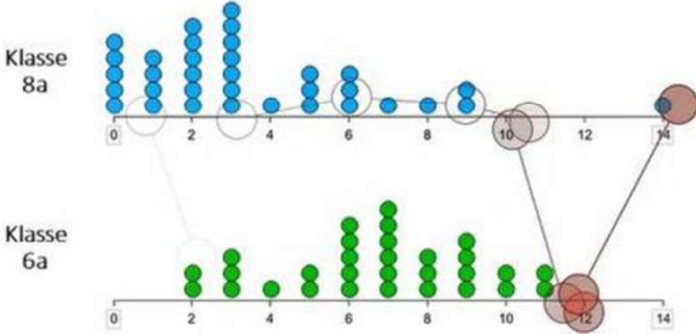
Oogbewegingen leerlingen
stippendiagrammen

Groep 8



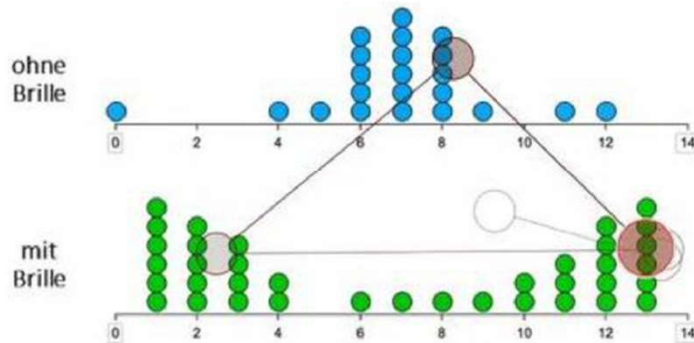
Schreiter & Vogel (2023). Eye-tracking measures as indicators for a local vs. global view of data. *Frontiers in Education*.

Groep 8

Screenshot of gaze plot	Related excerpt from the students' comments	Assigned categories (sub-categories)
	<i>“Here I looked who has the most games and saw that one person has 14 games and the others (from class 6a) have less...”</i>	Local (extreme values: maximum)

Schreiter & Vogel (2023).

Groep 8

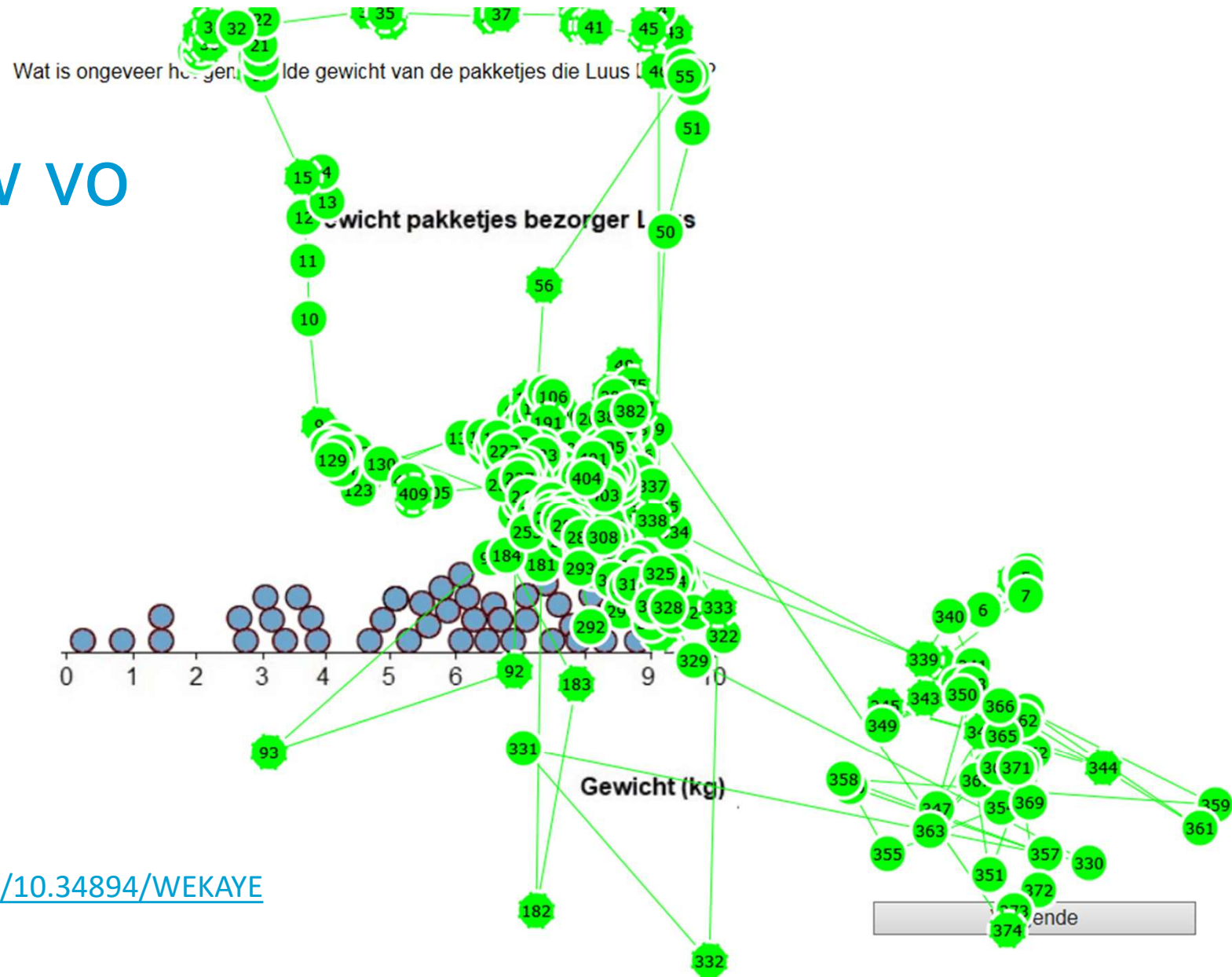


“I first looked at those without glasses and then I saw that they have relatively many in the middle but on the outside, that is, with more or less games, they have only a few people and then I looked at those with glasses and then I saw that they have fewer people in the middle but many with less or with a lot of games...”

Global (spread: division in three groups, majorit

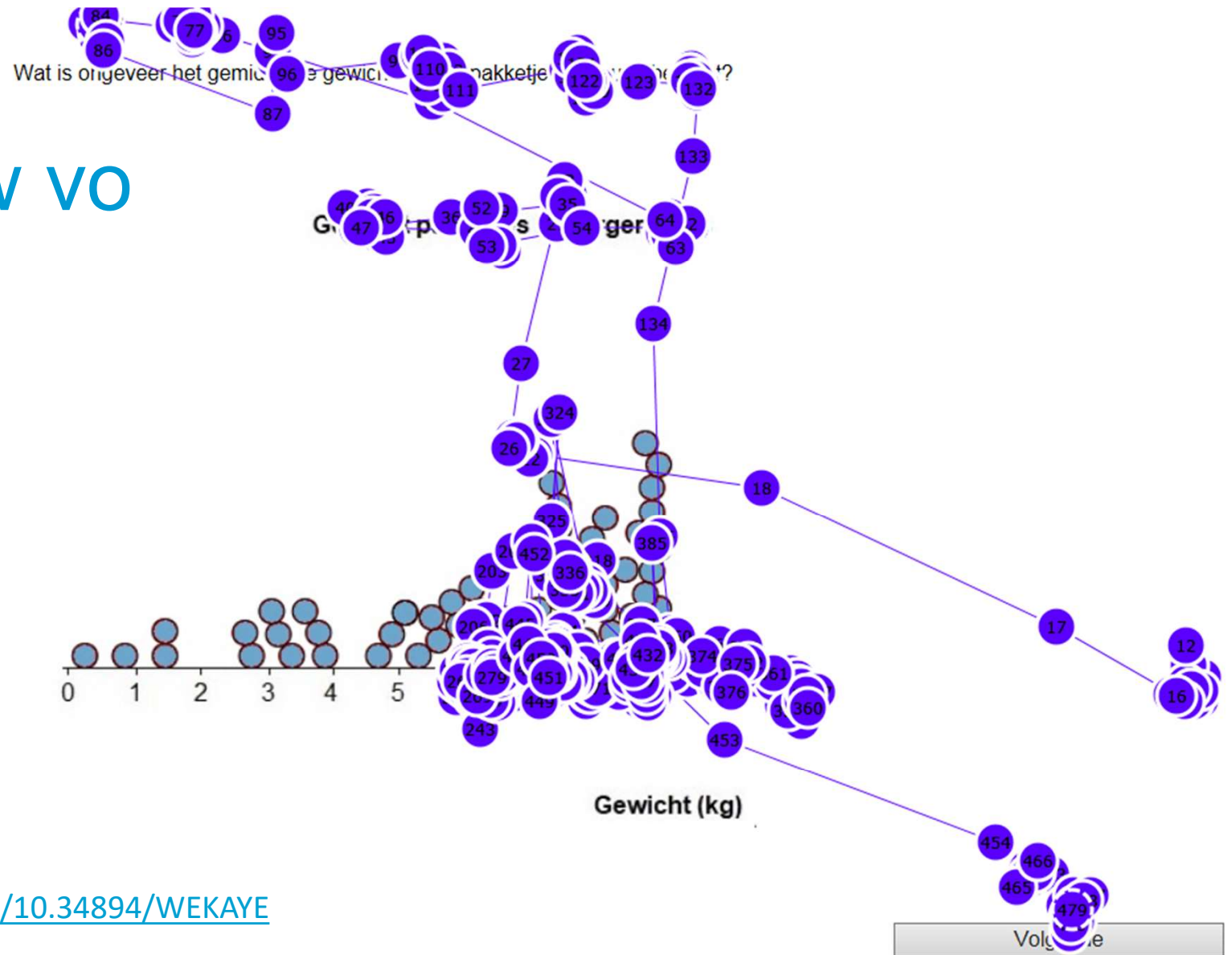
Schreiter & Vogel (2023).

Bovenbouw vo



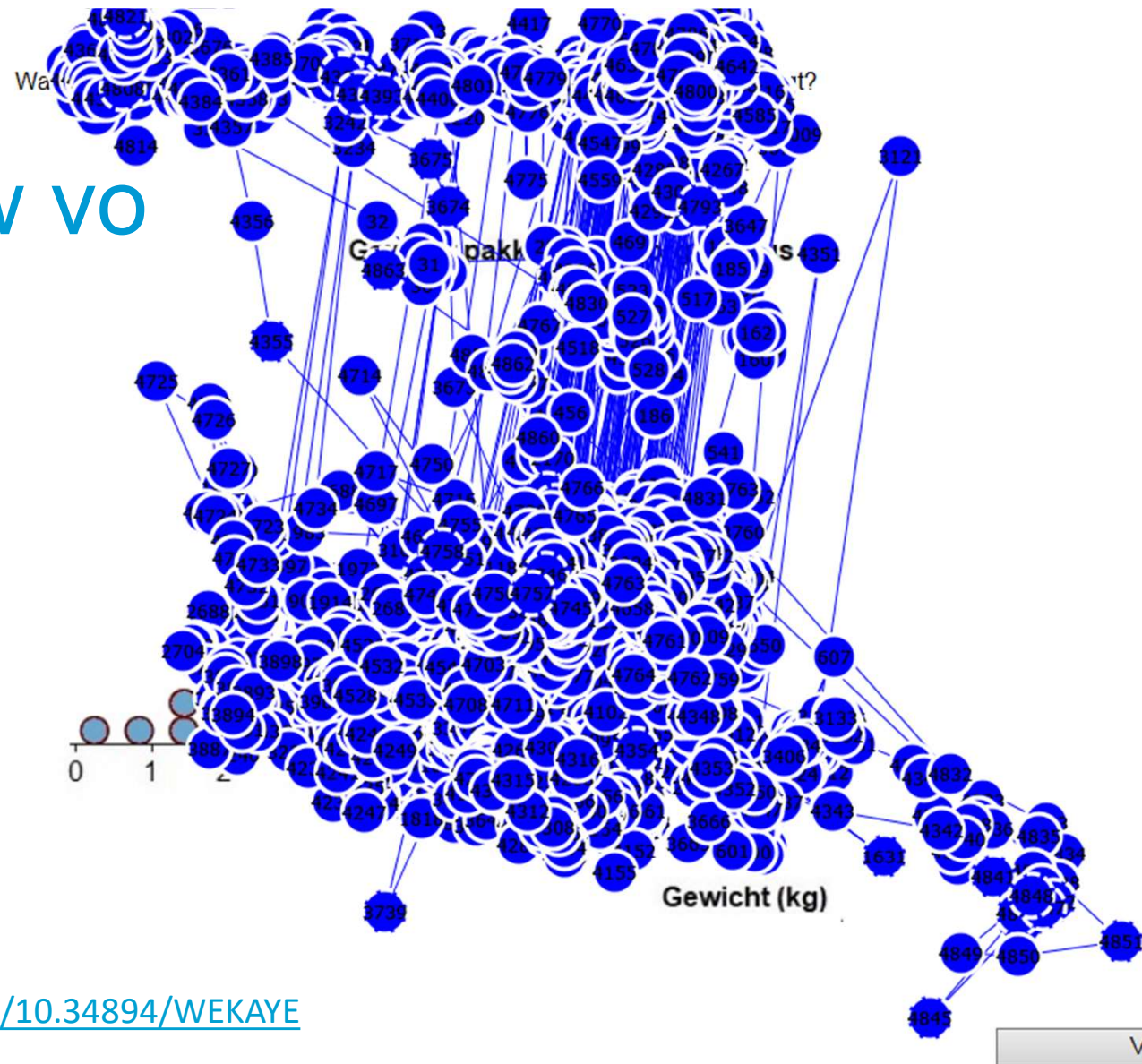
Boels, L. (2023). <https://doi.org/10.34894/WEKAYE>

Bovenbouw vo



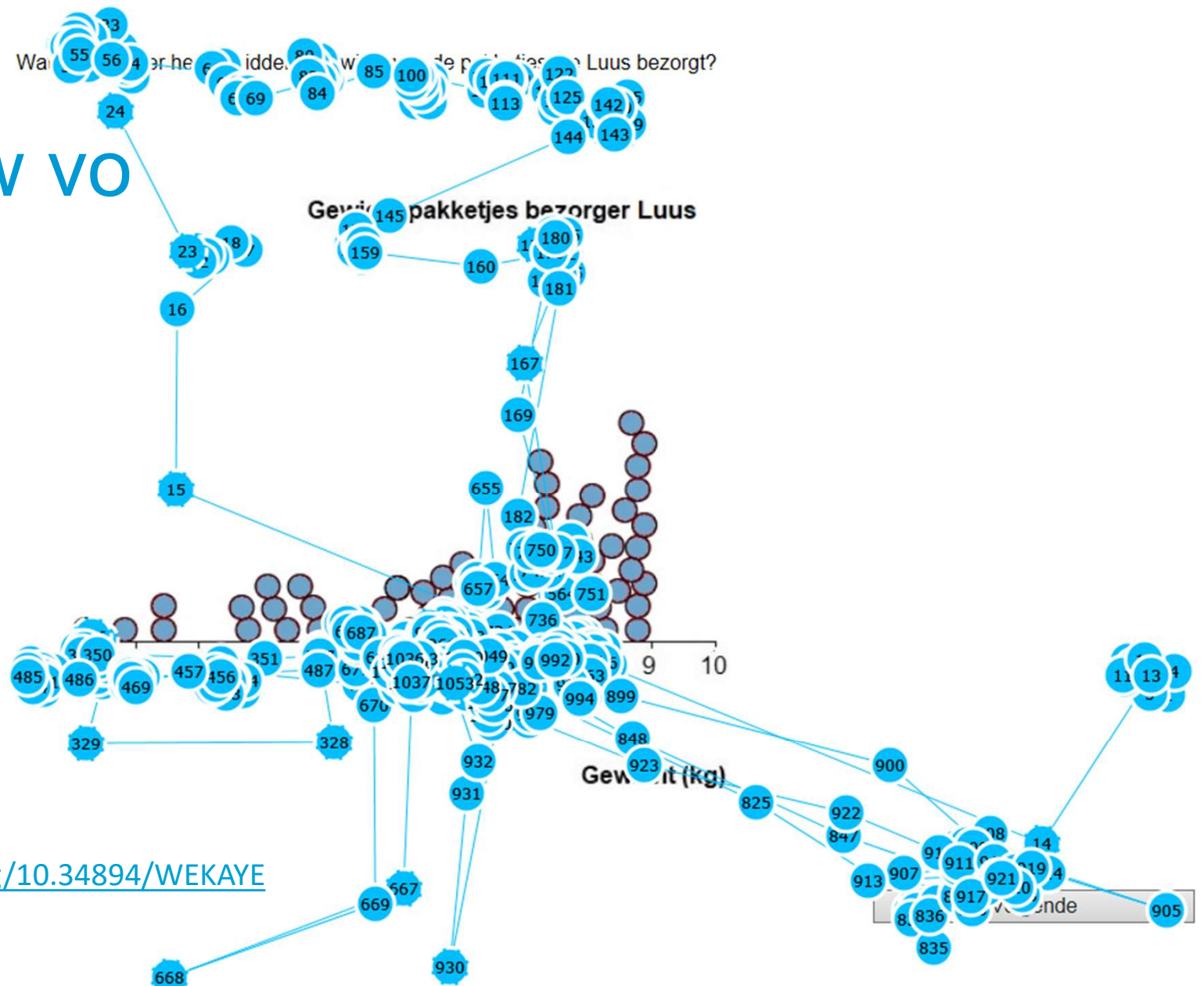
Boels, L. (2023). <https://doi.org/10.34894/WEKAYE>

Bovenbouw vo



Boels, L. (2023). <https://doi.org/10.34894/WEKAYE>

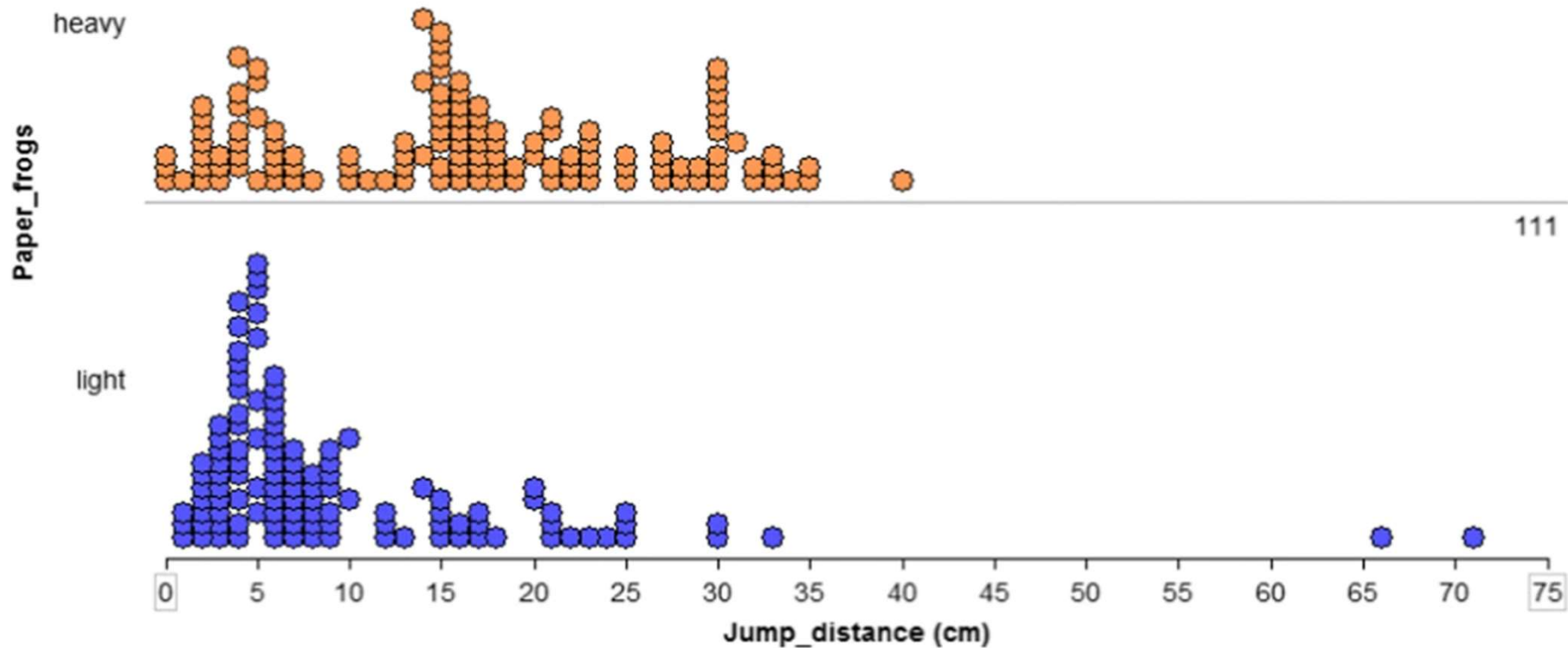
Bovenbouw vo



Boels, L. (2023). <https://doi.org/10.34894/WEKAYE>

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Taken leerlingen
Stippendiagrammen po



Distributions of long jump distances of paper frogs in *TinkerPlots*

Frischemeier (2019). Primary school students' reasoning when comparing groups using modal clumps, medians, and hatplots. *Mathematics Education Research Journal*, 31, 485–505.

Springende papieren kikkers.

Groep 6

Fielding-Wells & Hillman (2018). Supporting young students emerging understandings of centre through modelling. ICOTS10. https://iase-web.org/icots/10/proceedings/pdfs/ICOTS10_2B1.pdf?1531364242

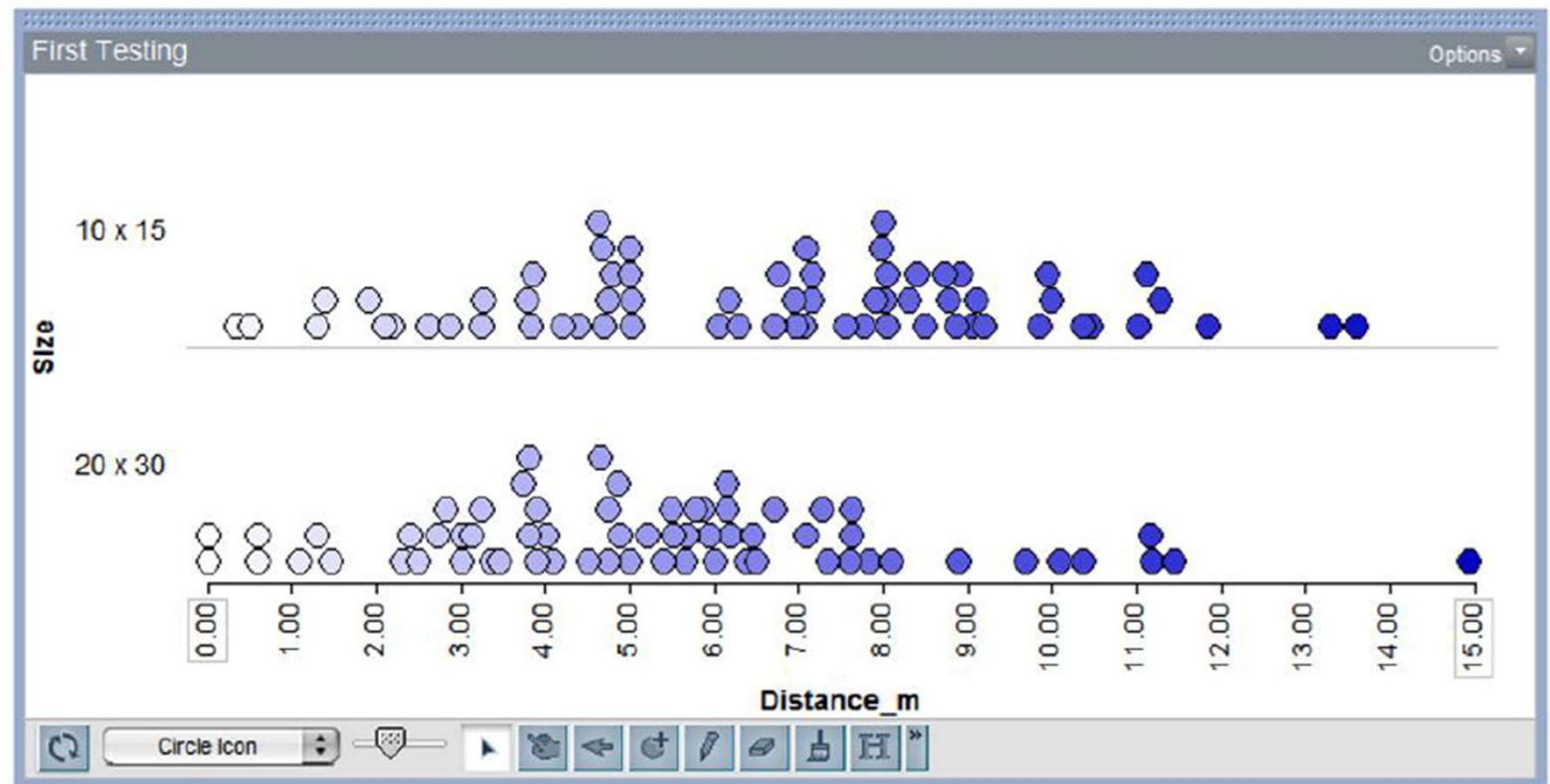


Fig. 1. TinkerPlots comparison of 10 x 15cm and 20 x 30cm flight test data

Welk papieren vliegtuig vliegt beter?
Gevouwen van 10 x 15 cm papier of van 20 x 30 cm papier

Groep 6 hatplot

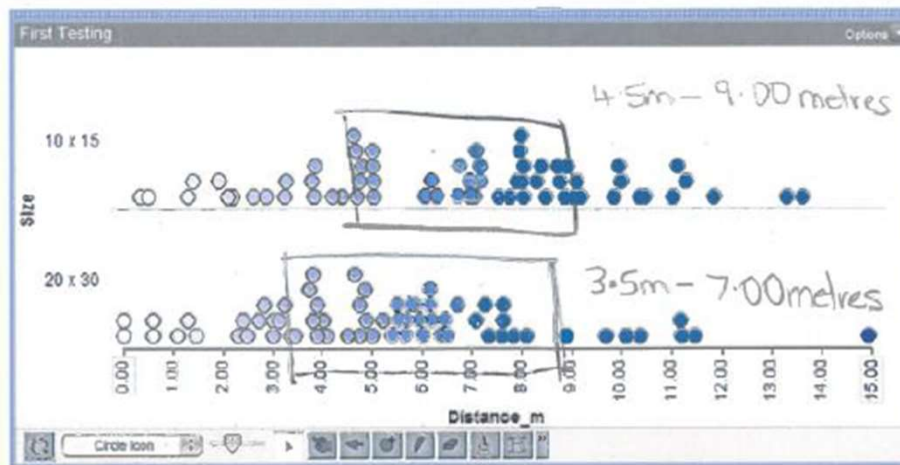


Fig. 2. Student identified centre (Janice)

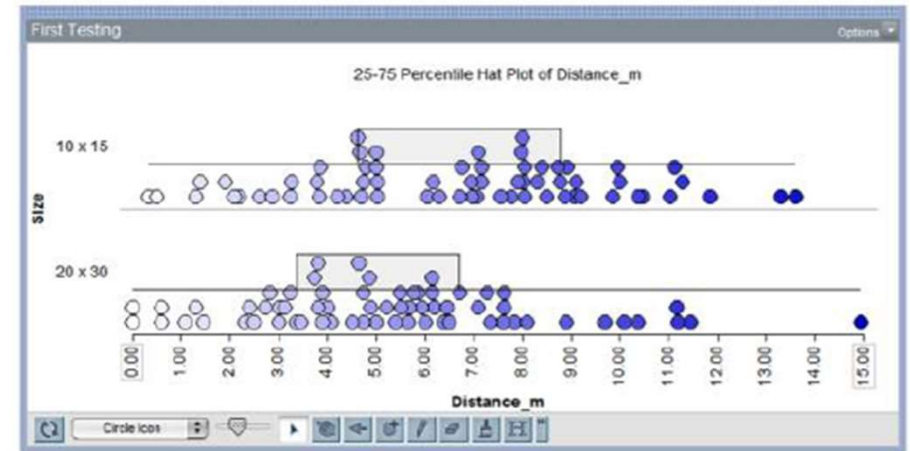
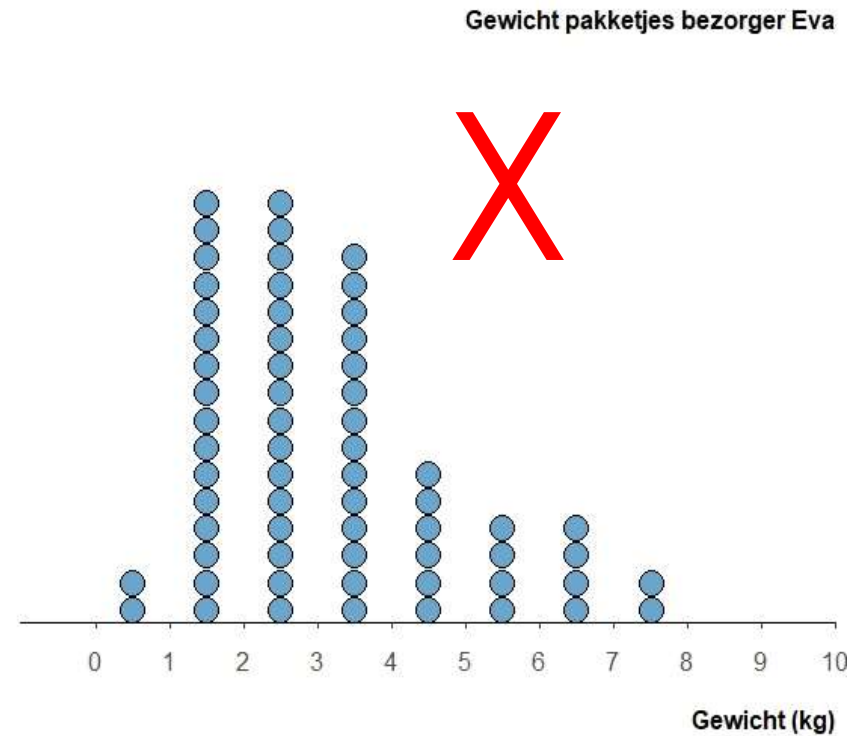
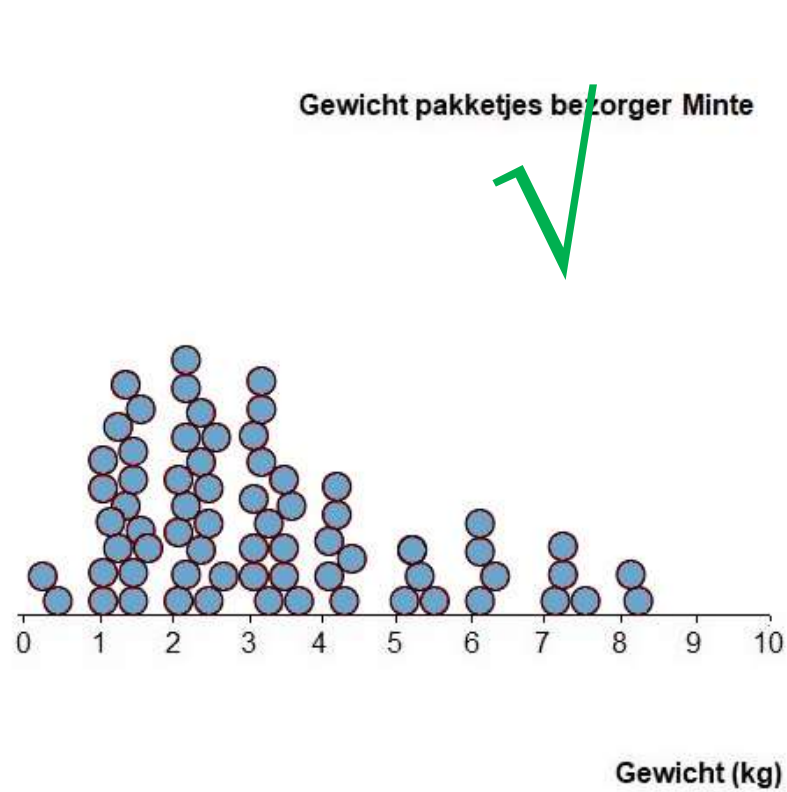


Fig 3. Comparing TinkerPlots Hats to students' hat plots

Fielding-Wells & Hillman (2018).

Welk papieren vliegtuig vliegt beter?
Gevouwen van 10 x 15 cm papier of van 20 x 30 cm papier

Resultaten oogbewegingsstudie vo (bovenbouw)



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Op weg naar histogrammen en boxplots in
het vo

Groep 6 dotplot $\rightarrow$ hatplot ($\rightarrow$ boxplot)

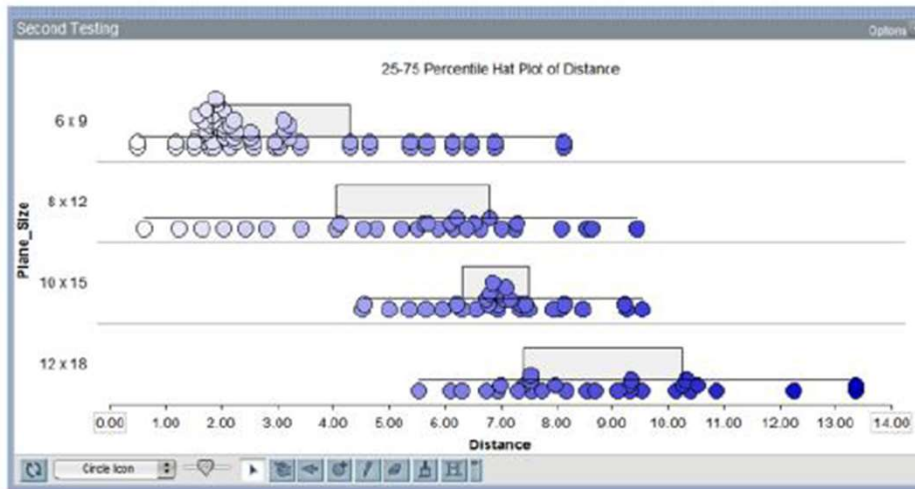


Fig. 4: Data from second round of testing comparing 6 x 9cm, 8 x 12cm, 10 x 15cm and 12 x 16cm paper sizes with hat plots

Fielding-Wells & Hillman (2018).

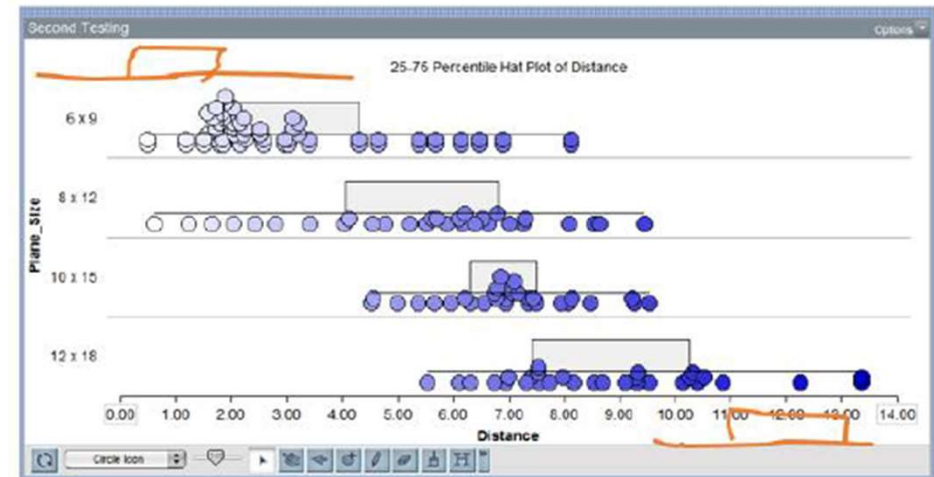


Fig. 5: Predicting the 'hats' for aeroplanes one size smaller and one size larger

Welk papieren vliegtuig vliegt beter?

Meer maten papier (links) en voorspellen andere maten (rechts)

vo hatplot → boxplot!

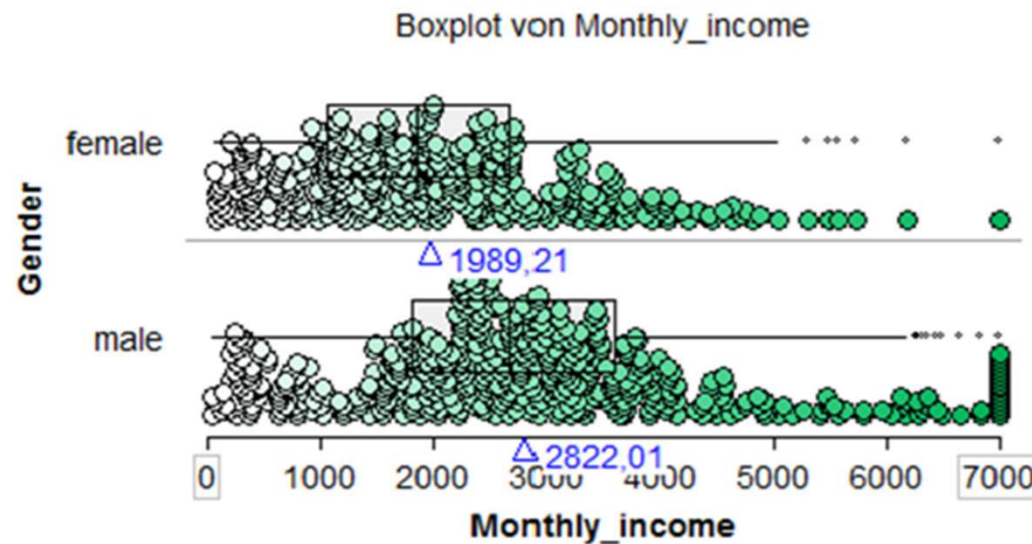
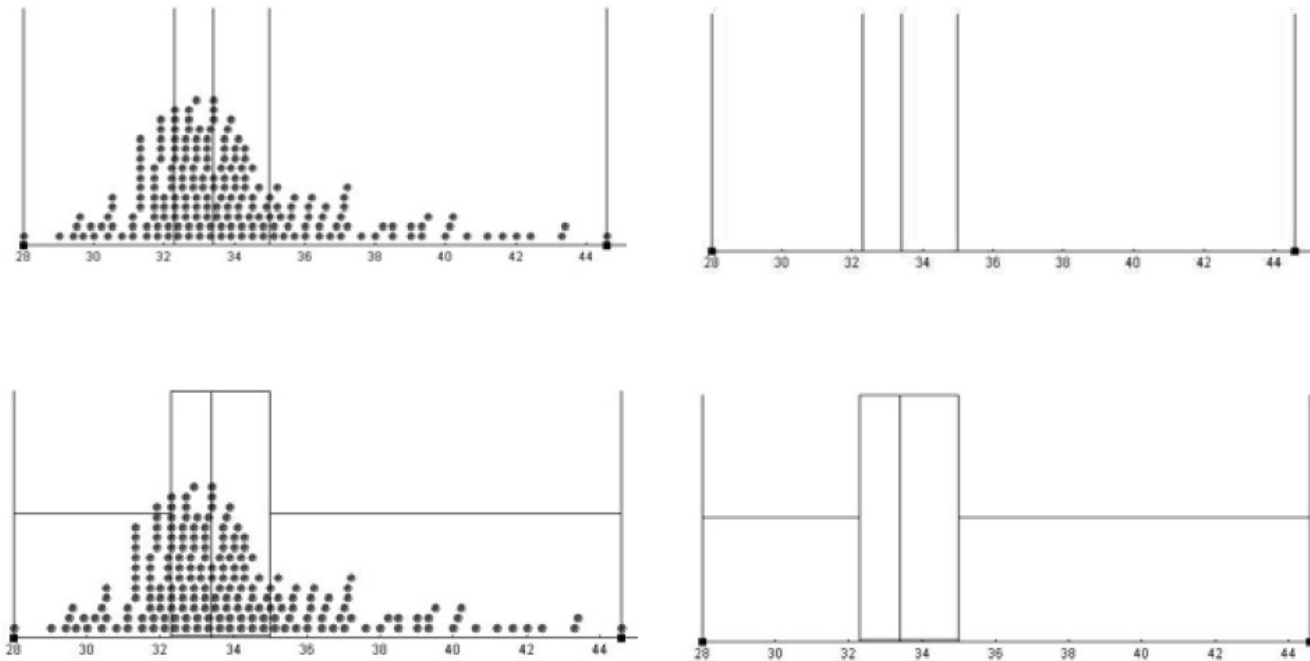


Figure 1: TinkerPlots Graph with boxplots and mean

Frischemeier & Biehler (2015). Preservice teachers' statistical reasoning when comparing groups facilitated by software. CERME9. <https://hal.science/hal-01287058>

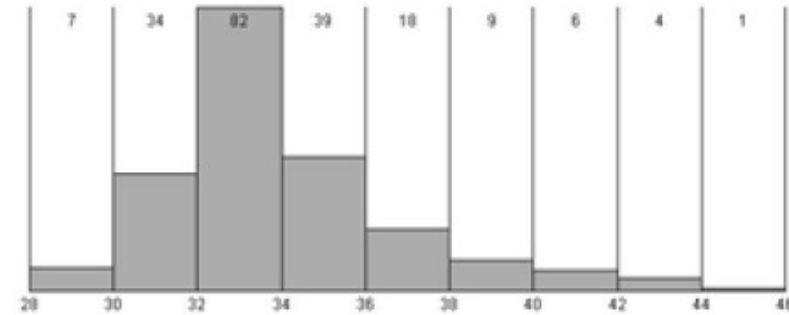
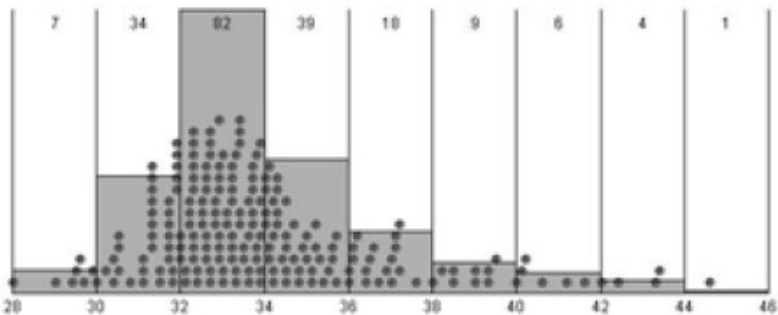
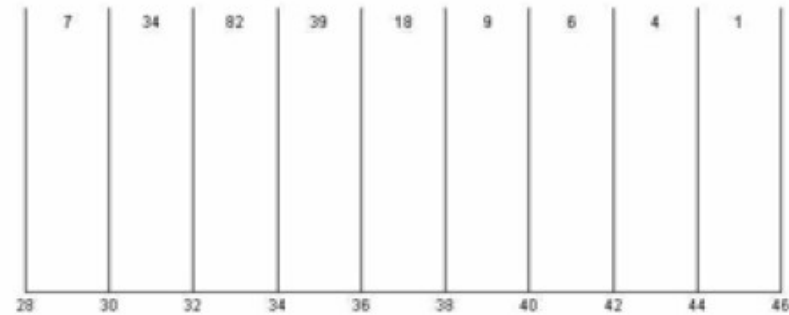
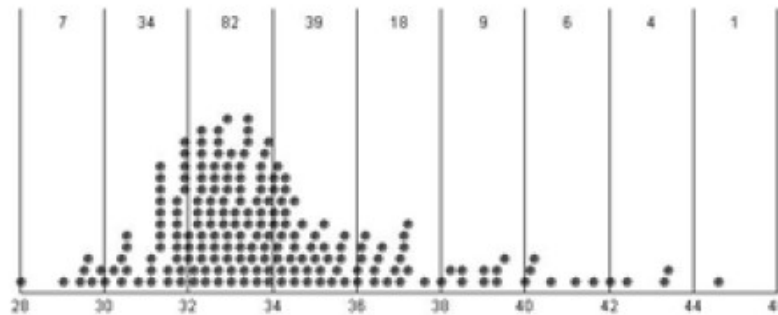
Welk papieren vliegtuig vliegt beter?
Meer maten papier (links) en voorspellen andere maten (rechts)

vo dotplot → hatplot → boxplot!



Bakker (2004).

vo dotplot → vaste intervallen → histogram!

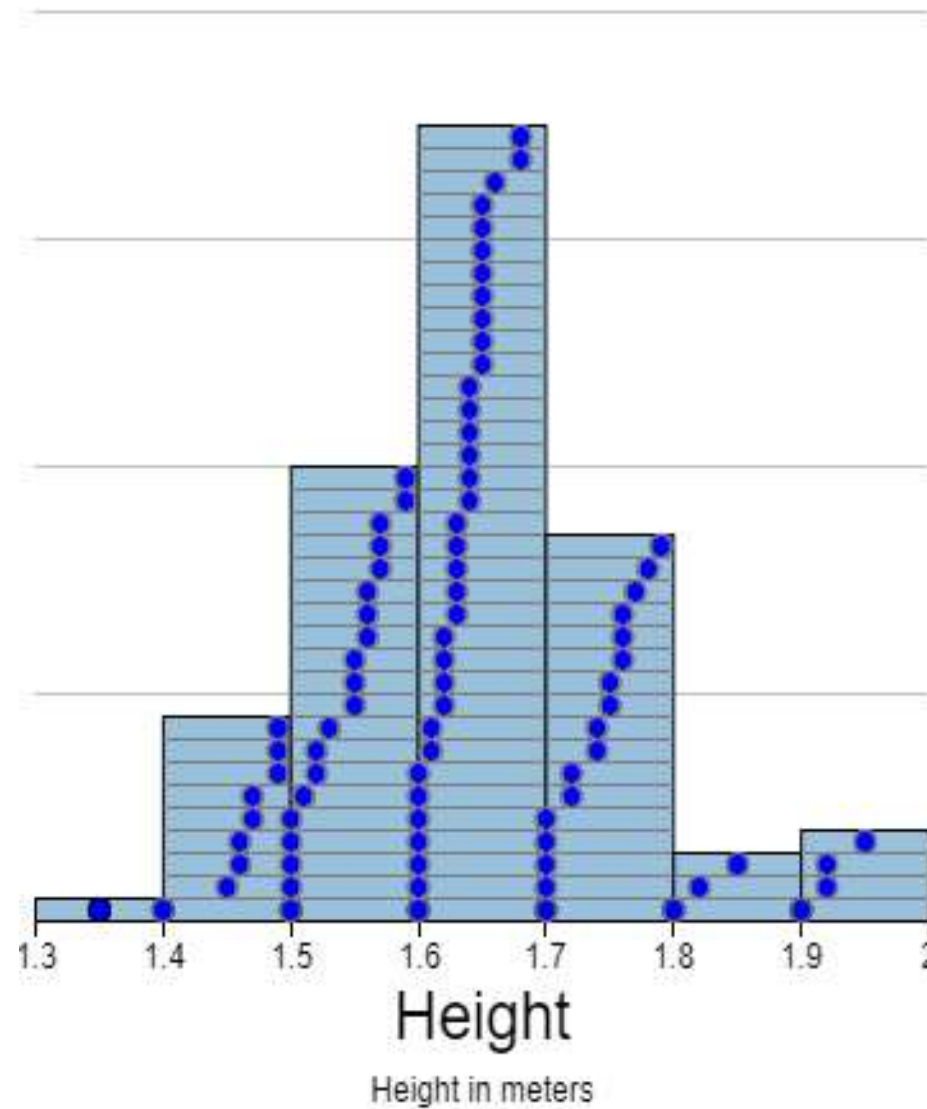


Bakker (2004).

Vermoeden

Histodot

Gemaakt met
VUstat.eu/apps



#.

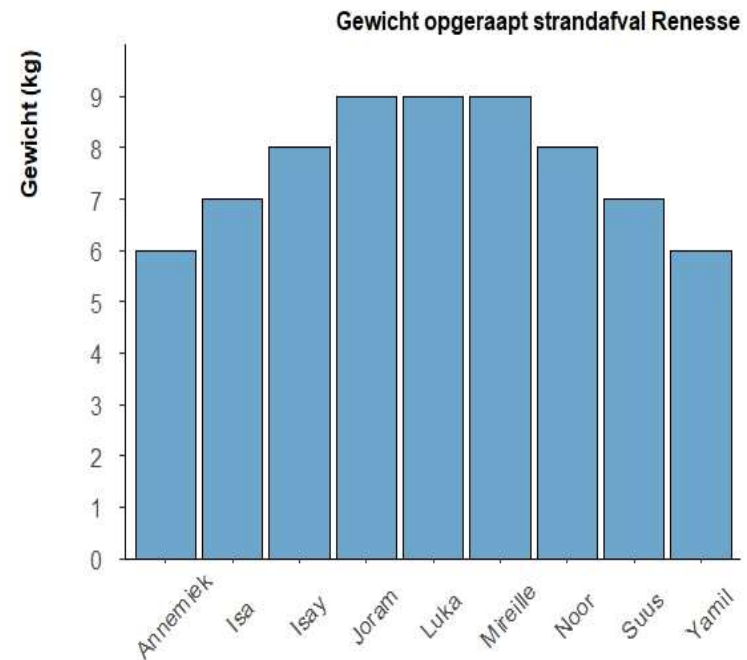
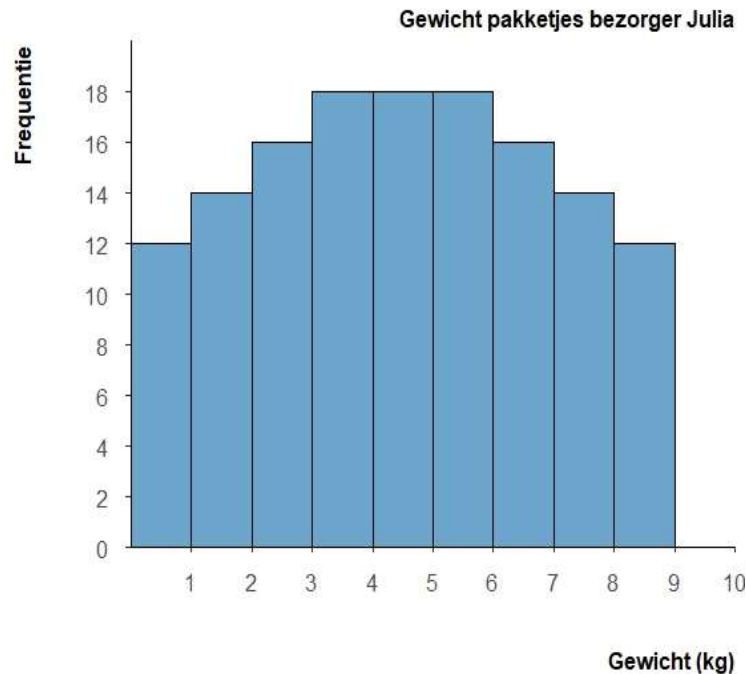
Nu even zelf

Wat geldt voor de spreiding/variatie van het gewicht in beide grafieken?

a) ongeveer gelijk b) linker meer c) rechter meer

Wat geldt voor het gemiddelde gewicht in beide grafieken?

a) ongeveer gelijk b) linker meer c) rechter meer



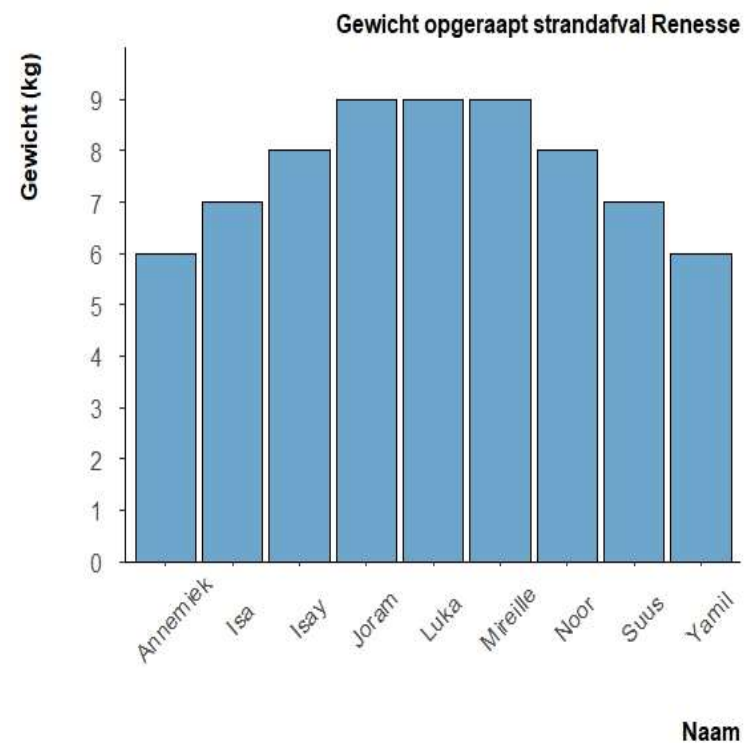
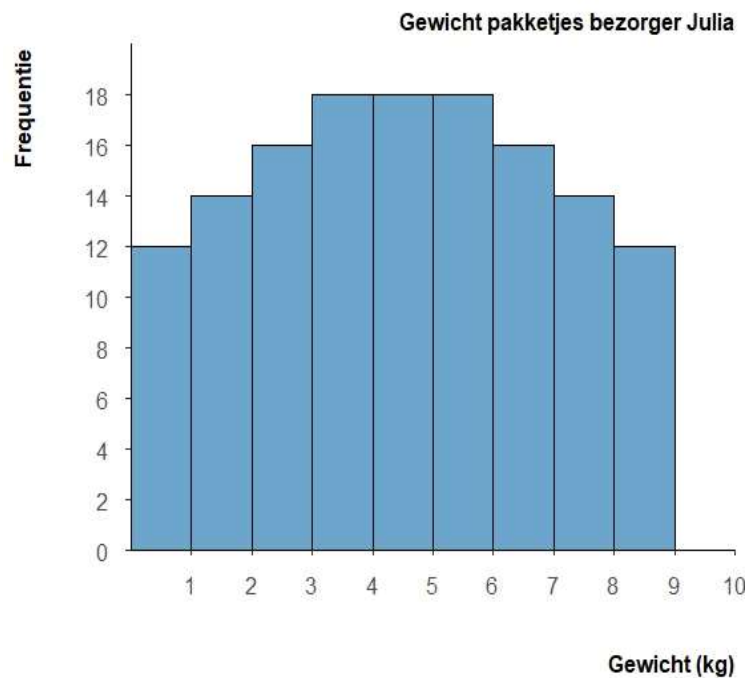
Naam

Wat geldt voor de spreiding/variatie van het gewicht in beide grafieken?
b) linker meer

Wat is ongeveer het gemiddelde gewicht in beide grafieken?

Links (histogram): 4,5 kg

Rechts (case-value plot): 7,7 kg



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Lesmateriaal lerarenopleiding

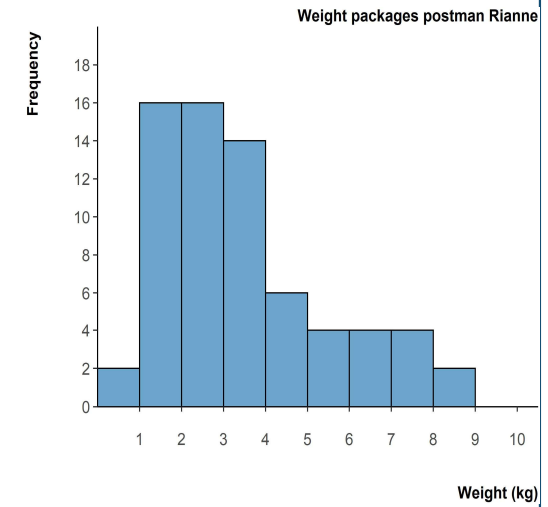
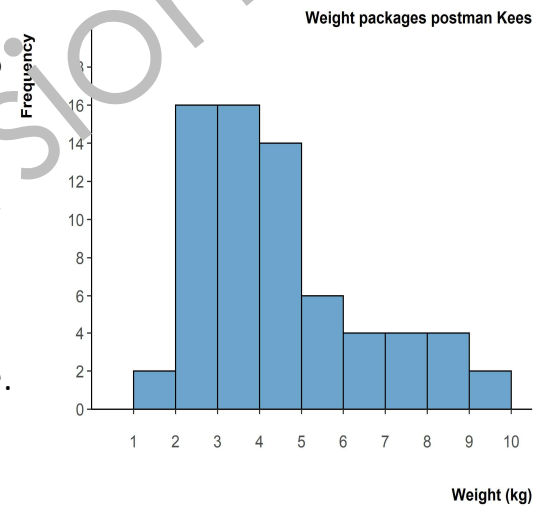
Imagine you are teaching a 10th-grade classroom on data interpretation, and you introduce your students to comparing data distributions using histograms.

You notice a student is having difficulties with a task you've assigned. The student makes a common error.

You will now see the task your student is working on.

Which postal worker delivers on average heaviest packages?

- a) Kees delivers on average the heaviest packages.
- b) Rianne delivers on average the heaviest packages.
- c) The mean weight for both is approximately the same.

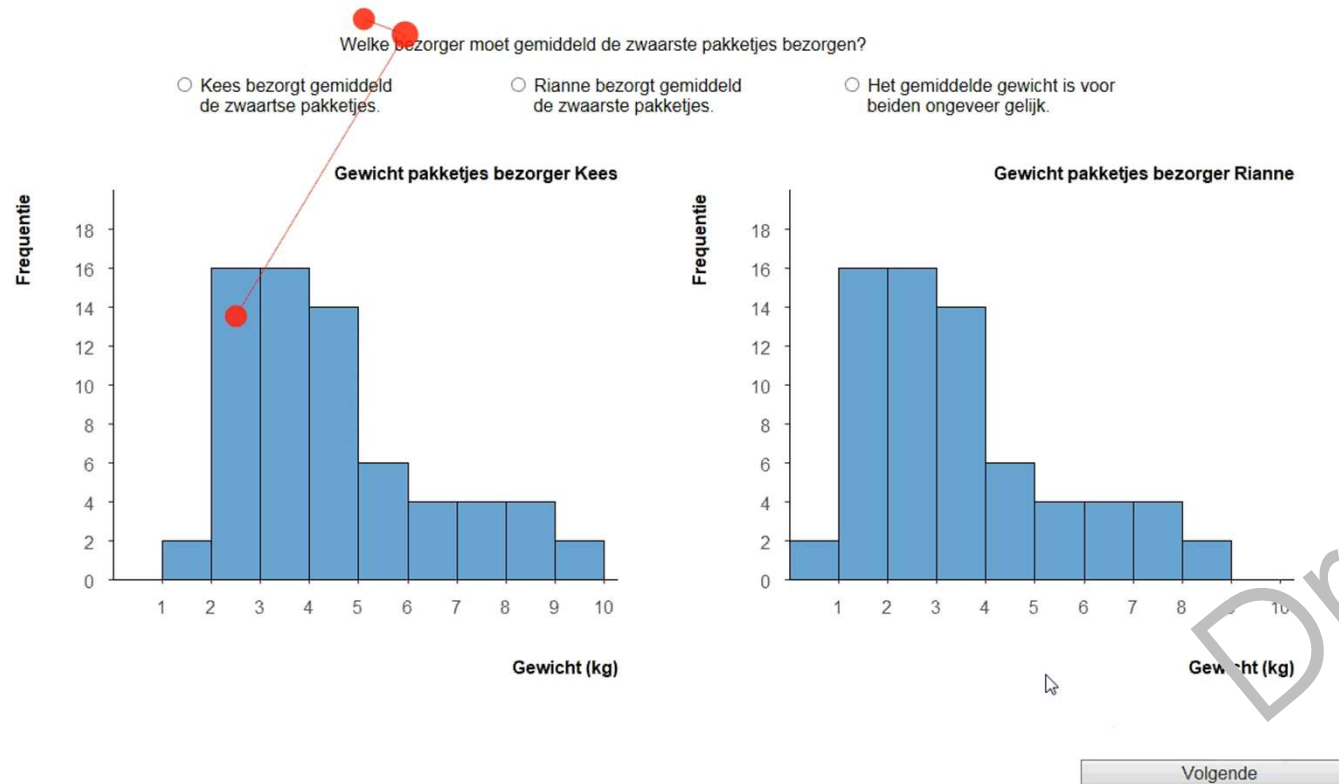


Work on the task and enter your solution.

Now imagine having access to eye-tracking technology.

You will now receive short gaze-overlaid video sequences. These show the eye movements of your student while Ali is working on the previously presented task. The eye movements reveal where the student's visual focus lies and provide insights into his/her thinking process.

Optional: click on more information on eye-tracking or on case-value plots (casus-staafdiagrammen) or on histograms, depending on the task at hand.



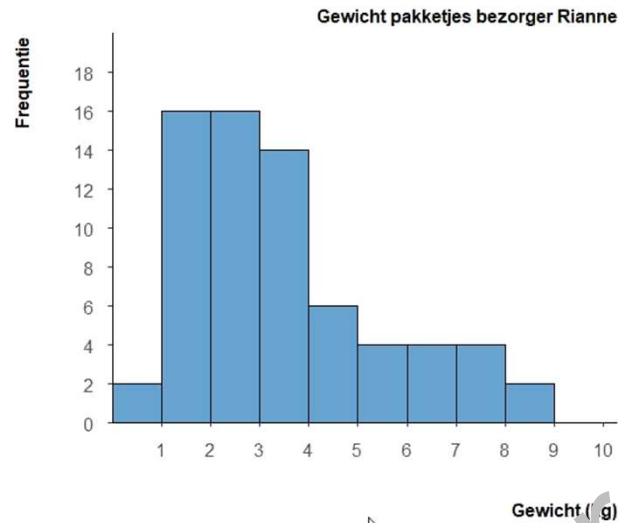
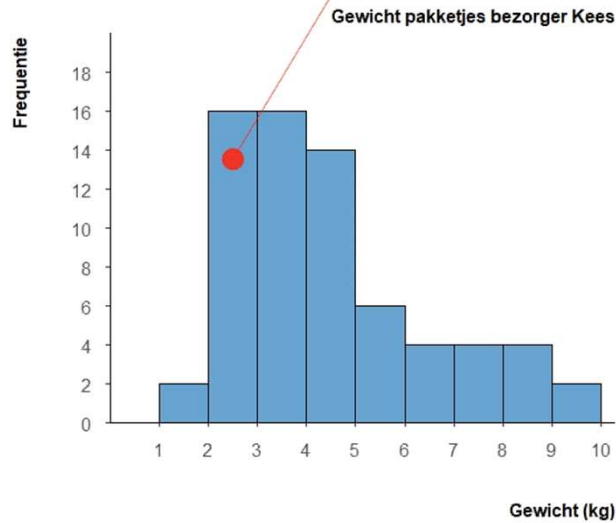
Have a look at the eyetracking video that show your student's solution process.

The student solution is
 "the mean weight for both is approximately the same."

[Click here to proceed](#)

Welke bezorger moet gemiddeld de zwaarste pakketjes bezorgen?

- ☐ Kees bezorgt gemiddeld de zwaarste pakketjes.
 ☐ Rianne bezorgt gemiddeld de zwaarste pakketjes.
 ☐ Het gemiddelde gewicht is voor beiden ongeveer gelijk.

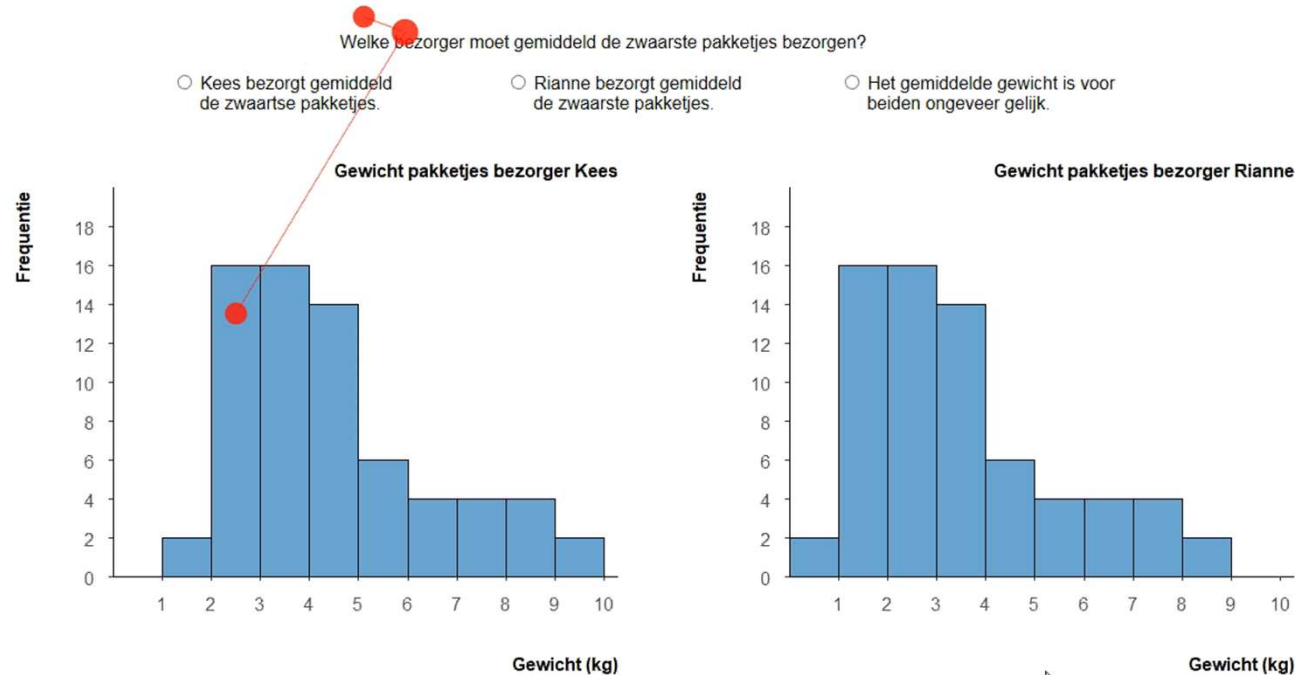


Now try to describe the student's strategy: What does he do? Try formulating it as precisely as possible.

The student solution is "the mean weight for both is approximately the same."

Volgende

[Click here to proceed](#)



You can now read the original comments of your student.

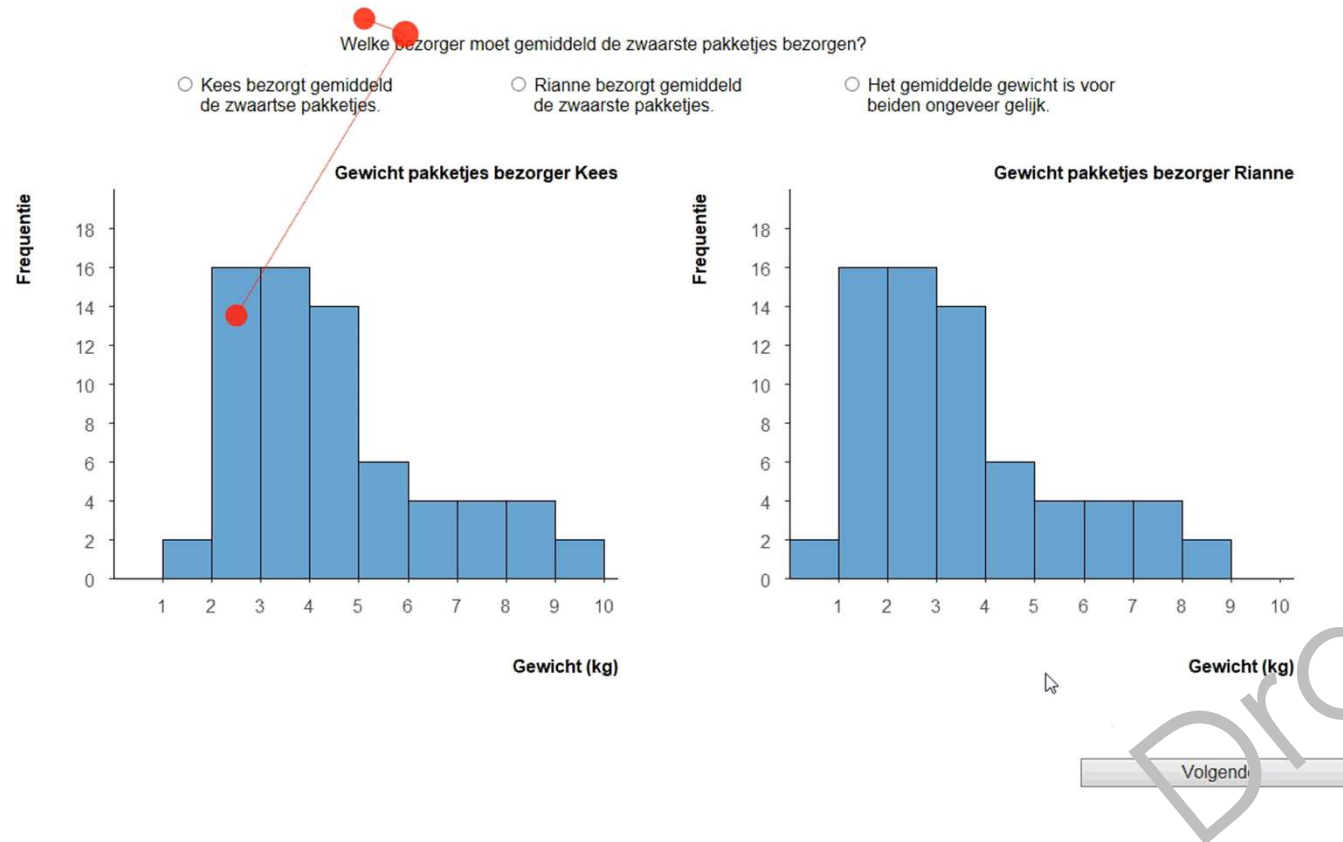
Indicate how well your assumptions on what the student did and thought align to those presented here.

Likert Scale

To a greater extent
Somewhat
Very little
Not at all

Volgende

"I noticed that the distributions were similar. I compared each pair of corresponding stacks across the histograms. They were the same. So, I believe the approximate means were the same."



Evaluate your student's solution process. What can you say about your student's understanding?

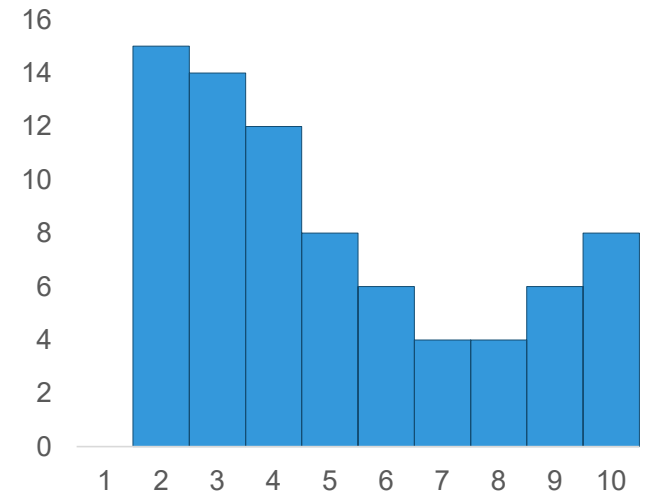
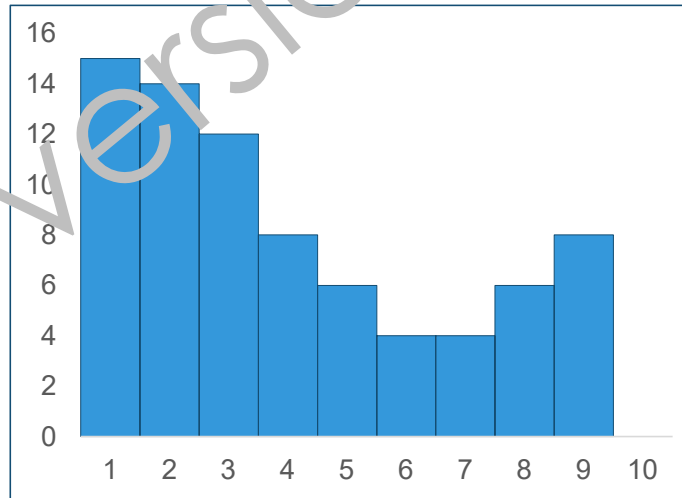
Open answer field

"I noticed that the distributions were similar. I compared each pair of corresponding stacks across the histograms. They were the same. So, I believe the approximate means were the same."

Imagine a student has the same systematic error, what could be the student answer to the following task?

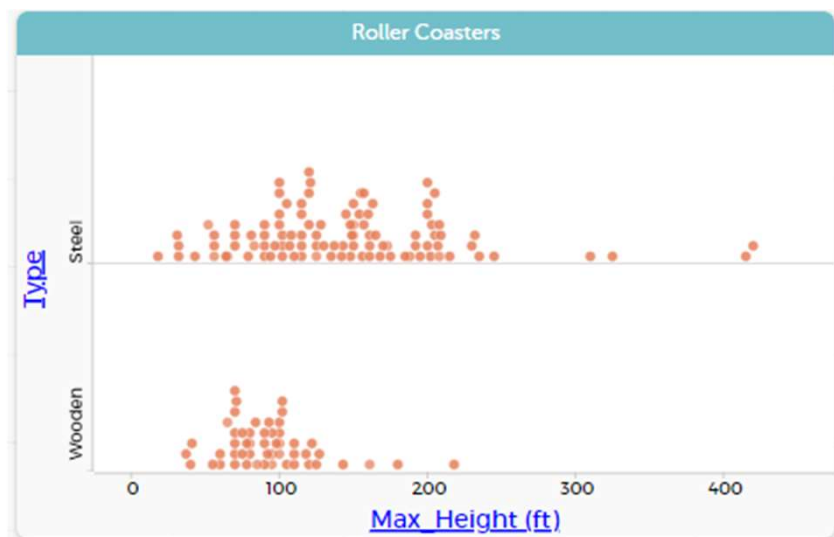
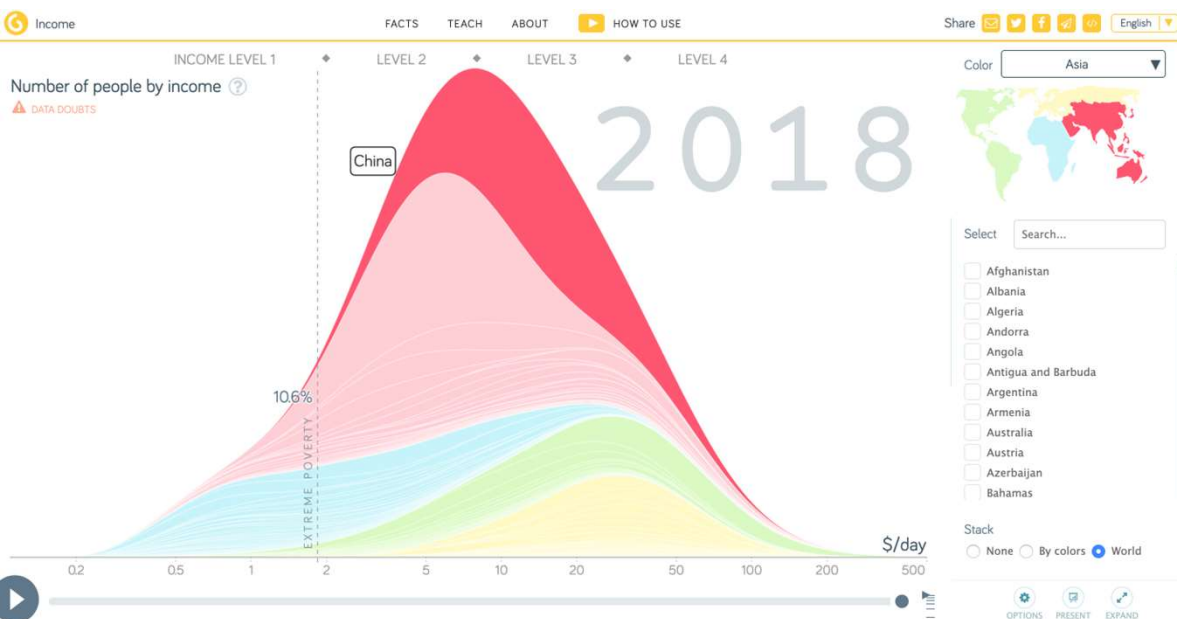
Which postal worker delivers on average the heaviest packages?

- a) Ellen delivers on average the heaviest packages.
- b) Titia delivers on average the heaviest packages.
- c) The mean weight is approximately the same for both.



Based on a known systematic error of applying a pairwise stack comparison strategy to histograms, describe the possible gaze patterns a student might take while comparing the histograms. What might their gaze patterns indicate about their solution process? Furthermore, explain how the student might verbally justify their conclusion that the histograms represent the approximately same mean weight.

#. Lesmateriaal



Lesmateriaal po



- Vustat: <https://www.vustat.eu/apps/> (histodot zit in de app data-analyse)
- <https://codap.concord.org/> (klik op try codap; is gratis; zie linksonder voor een afbeelding)
- <https://www.gapminder.org/> klik op resources. B.v.: dollarstreet! Of bubble chart.
- <https://www.tinkerplots.com/> Wordt niet meer onderhouden/geüpdated maar nog wel veel gebruikt in sommige landen/onderzoek. Codap is het webbased alternatief van dezelfde maker.

#.

Reclame

Ook lesmateriaal maar dan anders



Lesmateriaal gecijferdheid

- www.cenf.eu (Engelstalig; wordt momenteel vertaald; met docentenhandleiding en lesplan)
- <https://mbo.sites.uu.nl/category/situatie/> (Nederlandstalig; met werkbladen en lessuggesties)
- <https://www.examenblad.nl/> (b.v. vmbo-bb en kb opgaven wiskunde)

Meer lezen

- <https://gecijferdheidteltmee.nl/> (incl. voorbeeldmaterialen)
- <https://www.hu.nl/onderzoek/projecten/omgaan-met-hedendaagse-informatiebronnen> (rapport)
- <https://www.movisie.nl/publicatie/wat-werkt-versterken-gecijferdheid> (infographic)
- <https://www.telmeemettaal.nl/nieuws/praktijkverhalen/kees-hoogland> (interview)
- <https://epale.ec.europa.eu/nl/blog/ontwikkelingen-op-het-gebied-van-gecijferdheid-als-basisvaardigheid> (opinie)

Bustijden

maandag t/m vrijdag	zaterdag	zon- en feestdagen
06 40 50	06 27 57	06 27 57
07 00 10 17 27 37 47 57	07 27 57	07 27 57
08 07 18 28 38 50	08 27 57	08 27 57
09 00 10 20 30 40 50	09 27 57	09 27 57
10 00 10 20 30 40 50	10 27 57	10 27 57
11 00 10 20 30 40 50	11 27 57	11 27 57
12 00 10 20 30 40 50	12 28 58	12 27 57
13 00 10 20 30 40 50	13 28 58	13 27 57
14 00 10 20 30 40 50 58	14 28 58	14 27 57
15 00 10 20 30 40 50	15 28 57	15 27 57
16 05 15 25 35 45 55	16 27 57	16 27 57
17 05 15 25 36 46 56	17 27 57	17 27 57
18 11 26 41 57	18 27 57	18 27 57
19 12 27 42 57	19 27	19 27
20 12 27 42 57	20 27	20 27
21 12	21 27	21 27

Ga naar
werkblad

"Ja, als ik bijvoorbeeld om tien uur weg wil en dan zie ik hem hier om tien uur, tien over tien, 20, zo zie ik hem, dat die elke tien minuten gaat. Zo lees ik hem en dan moet je natuurlijk effe goed opletten dat je de juiste blokken hebt."

Eens per maand beschrijven wij een alledaagse situatie waarin rekenen/wiskunde

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HIER KOMT ALLES SAMEN