Supporting All Students to be Creative, Analytical Thinkers in the Mathematics Classroom Through Effective, Research-Based Teaching Practices

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Overview of Session

- Problem Solving & Critical Thinking
- Benefits of engaging students in problem solving
- Productive Struggle-Persistence & Perseverance
- Solve Problems-Examine Student Solutions
- Connect to Effective Mathematics Teaching Practices & Mathematical Practices for our Students

Guiding Questions

- How does problem solving enhance creativity and support critical thinking and why is it important?
- What are effective mathematics teaching practices that enhance student thinking and engage students in problem solving?
- What mathematical practices do we want our students to develop?
- What is the role of productive struggle in teaching and learning maths?

What word comes to mind when you think of problem solving and creativity in teaching and learning mathematics?

What does it mean to engage children in critical thinking in mathematics ?

Turn & Talk

Why Problem Solving?

- Supports making connections across disciplines and supports future professional opportunities
- Supports students positive mathematical identity
- Matter of Equity-access
- Builds confidence, persistence, flexibility, creativity, perseverance, communication, curiosity
- Gives student voice, promotes discussion
- Shifts math authority to student

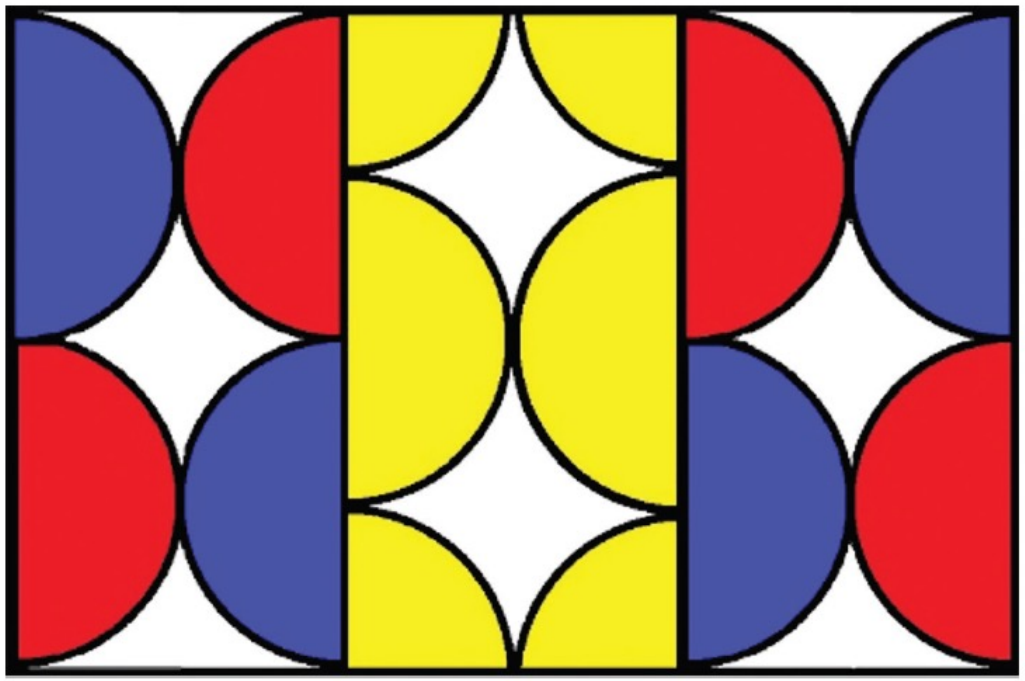
NCTM 2000; 2014, 2018

Why Problem Solving?

Children experience joy when they are encouraged to be creative and are provided with mathematical choices, such as generating their own approaches and strategies for solving mathematical problems.
(NCTM 2020, Catalyzing Change, p. 21)

What do you notice?
What do you wonder?

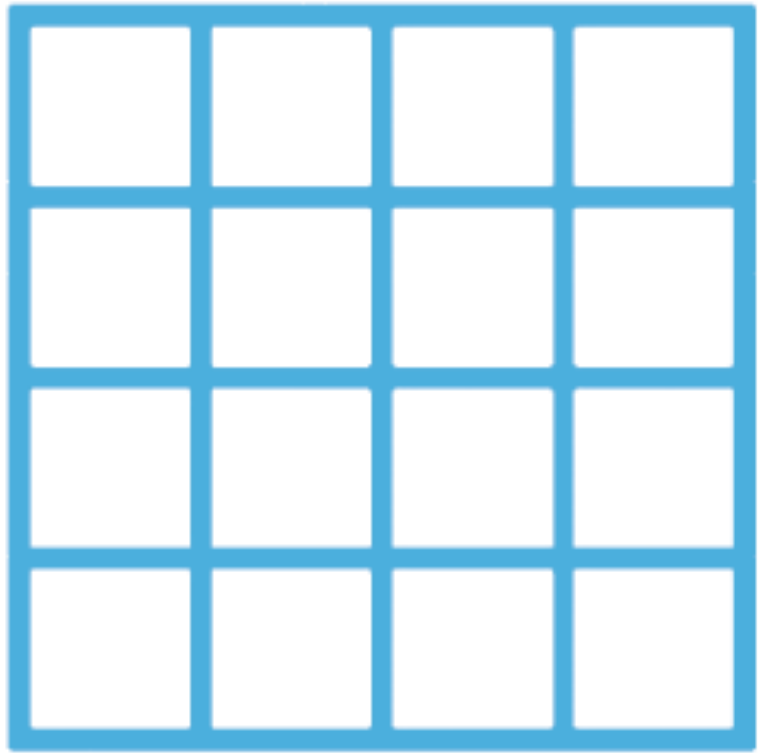
Problems to Ponder, December 2020 MTLT, p. 1049



https://www.nctm.org/noticeandwonder/

10

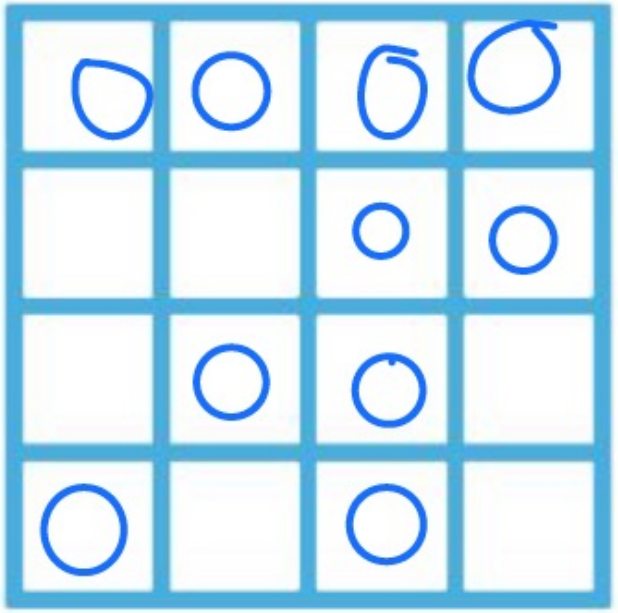
Place 10 circles in the grid, so that each row, each column, and each diagonal has an even number of circles.



MTLT, December 2021
Problems to Ponder
p. 970

Student Solutions

1



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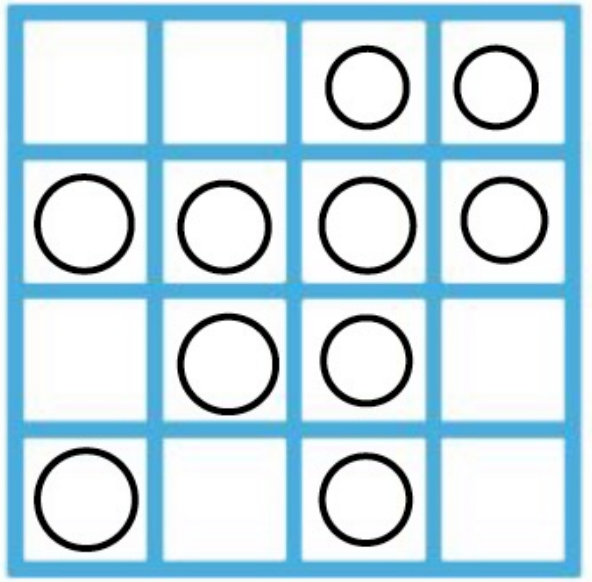

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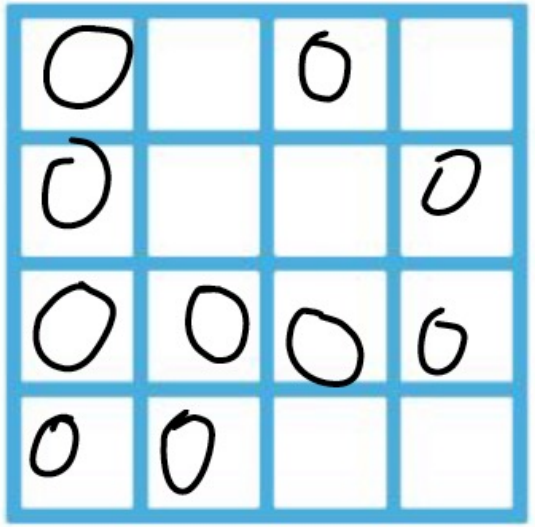
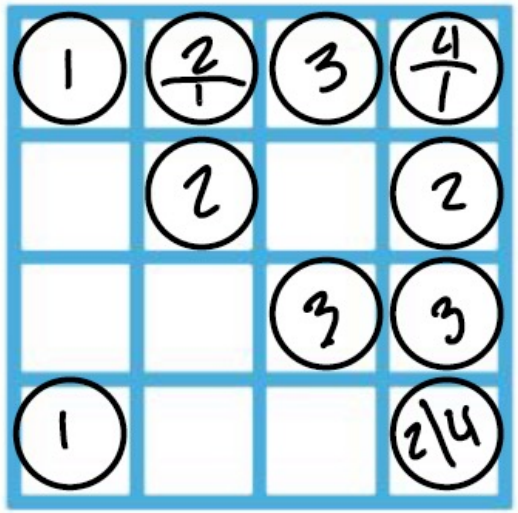


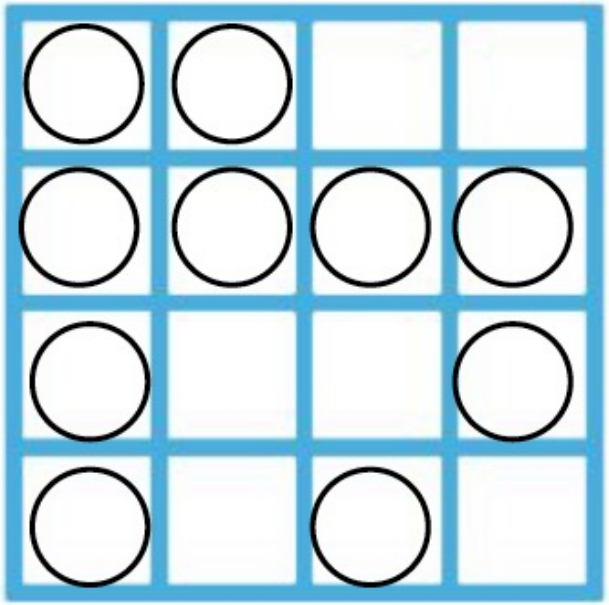


4

10

Place 10 circles in the grid, so that each row, each column, and each diagonal has an even number of circles.

same



2, 4

16 > 10

4 + (3 x 2) = 10

6 empty squares

Can't have 3

Why Problem Solving?

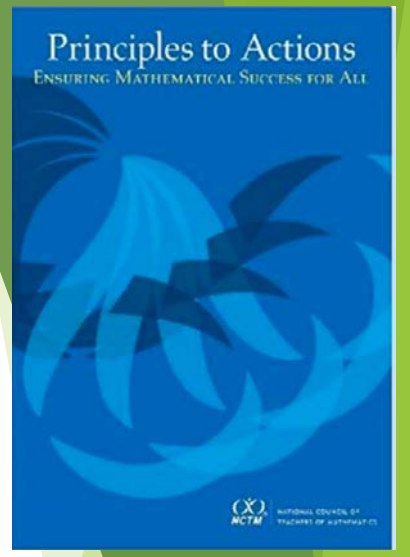
Classroom environments in which students are provided opportunities to engage in mathematical investigation, communication, and group problem solving, while also receiving feedback on their work from both experts and peers, have a positive effect on learning.

(CBMS, 2016, p. 1)

Mathematics Teaching Practices

(NCTM, PtA 2014, p. 10; 2024) (p. 4 of Executive Summary)

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.



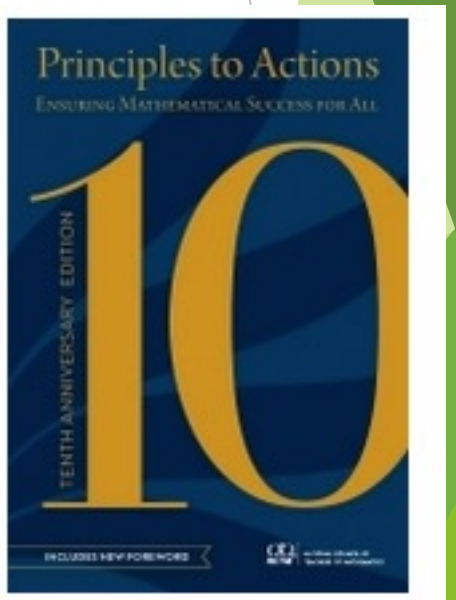


Table 1 Characterizing Students' Productive Struggle

Productive Struggle *Is*	Productive Struggle *Is Not*
Using existing understandings to engage with problems that do not have immediately apparent solutions	Waiting for information to be presented so it can be memorized or practiced
Persevering in making sense of mathematics during problem solving	Feeling despair because the mathematics makes little sense
Solving problems and grappling with key mathematical ideas that are within reach	Experiencing needless frustration or extreme levels of challenge with overly difficult problems

Note: Adapted from Hiebert and Grouws (2007).

3 Take-aways

1. Select a mathematics problem that allows for productive struggle
2. Value and support productive struggle
3. Reflect on our efforts to promote productive struggle so that we can improve

Productive Struggle in Action by Baker, Jessup, Jacobs, Empson, & Case. *Mathematics Teacher: Learning and Teaching PK-12* 113 (5). 361-367

11

Place the digits 1 to 9, once and one per box, so that the following statements are true:

- The boxes containing the 1, the 2, and all the digits between them have a product of 72.
- The boxes containing the 2, the 3, and all the digits between them have a product of 336.
- The boxes containing the 3, the 4, and all the digits between them have a product of 1,344.
- The boxes containing the 4, the 6, and all the digits between them have a product of 1,080.

MTLT, December 2021 Problems to Ponder p. 971

11

Place the digits 1 to 9, once and one per box, so that the following statements are true:

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- The boxes containing the 4, the 6, and all the digits between them have a product of 1,080.

$1 \cdot 2 = 2 \quad 72 \div 2 = 36$

$2 \cdot 3 = 6 \quad 336 \div 6 = 56$

$3 \cdot 4 = 12 \quad 1344 \div 12 = 112$

$4 \cdot 6 = 24 \quad 1080 \div 24 = 45$

$$
\begin{array}{r}
112 \\
12\overline{)1344} \\
-12 \\
\hline
14 \\
-12 \\
\hline
24 \\
-24 \\
\hline
0
\end{array}
\qquad
\begin{array}{r}
45 \\
24\overline{)1080} \\
-96 \\
\hline
120 \\
-120 \\
\hline
0
\end{array}
$$

6	5	1	9	4	2	7	8	3

11

Place the digits 1 to 9, once and one per box, so that the following statements are true:

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3	8	7	2	4	9	1	5	6

$3 \times 8 \times 7 \times 2 = 336$
$2 \times 4 \times 9 = 72$
$3 \times 8 \times 7 \times 2 \times 4 = 1344$
$4 \times 9 \times 5 \times 6$

Closer together

Farther

$2\overline{)72}$ → 36

$18\overline{)72}$ → 6, 1, 2

5 between 4, 6

42

168 5789

168

336

14 × 3 = 42

÷4 × 3 = 12

$3\overline{)336}$ → 112

$7\overline{)112}$ → 14, 7, 1, 42

11

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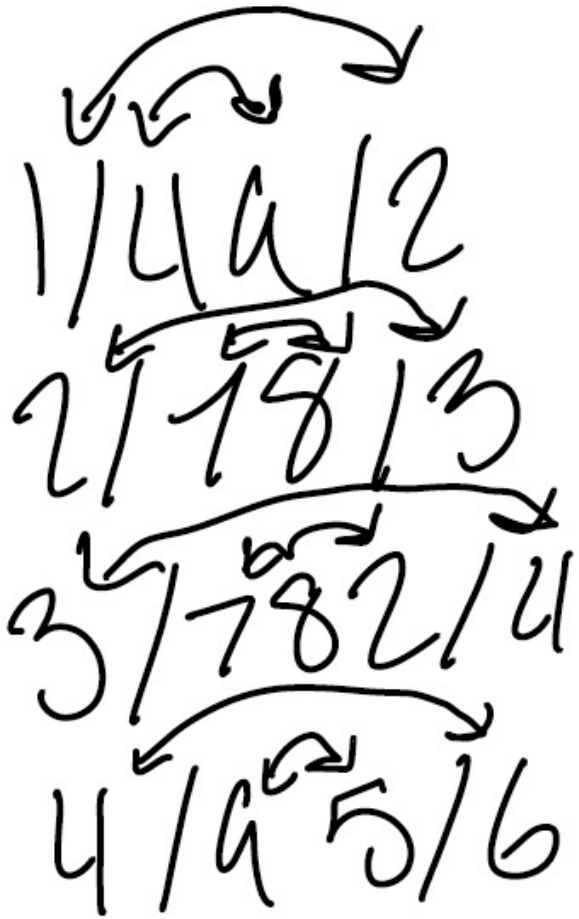
3	8	7	2	4	9	1	5	6



3	8	7	2	4	9	1	5	6

6	5	1	9	4	2	7	8	3

What do you notice?
What do you Wonder?
Are there other solutions?
How many unique solutions?

11 There are four solutions:

3,7,8,2,4,9,1,5,6

3,8,7,2,4,9,1,5,6

6,5,1,9,4,2,7,8,3

6,5,1,9,4,2,8,7,3

Mathematical Practices For Students

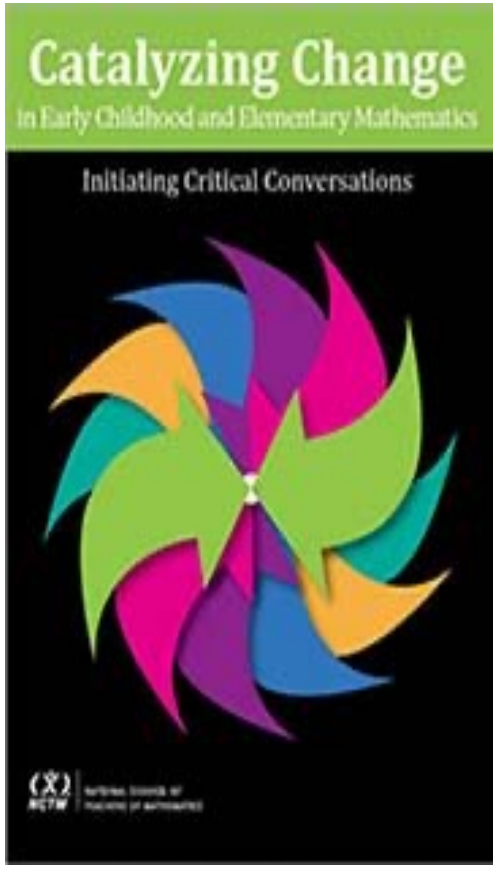
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CCSS-M, 2010 https://www.thecorestandards.org/Math/Practice/

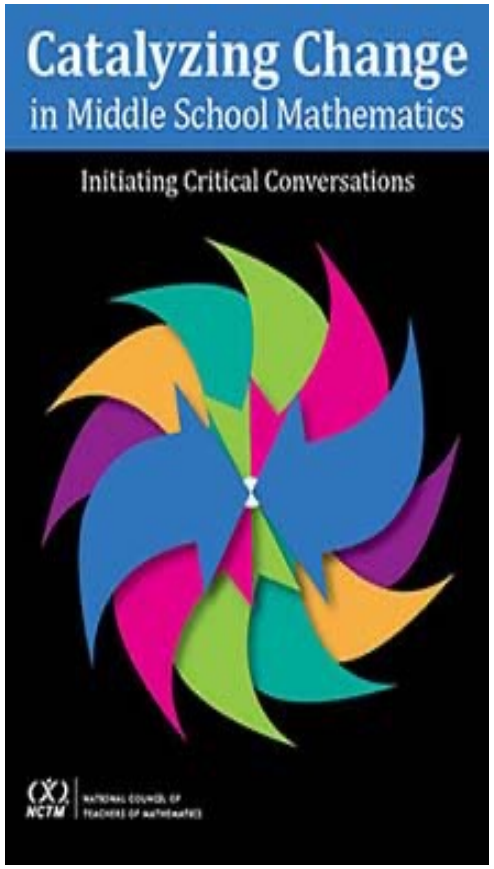
Catalyzing Change Series

www.nctm.org/change

2020



2020



2018

Research and Practice-The Connections

Four Key Recommendations

Recommendation 1

Broaden the Purposes of Learning Mathematics

Recommendation 2

Create Equitable Structures in Mathematics

Recommendation 3

Implement Equitable Mathematics Instruction

Recommendation 4

Develop Deep Mathematical Understanding

Recommendation #1

Broaden the Purposes of Learning Mathematics

Develop Deep Mathematical Understanding as Confident and Capable Learners

Understand and Critique the World Through Mathematics

Experience the Wonder, Joy, and Beauty of Mathematics

Recommendation #3

Implement Equitable/Effective Mathematics Instruction

Consistent with research-informed equitable, effective teaching practices

Nurture & foster student's positive math identity and strong sense of agency

Quality over quantity

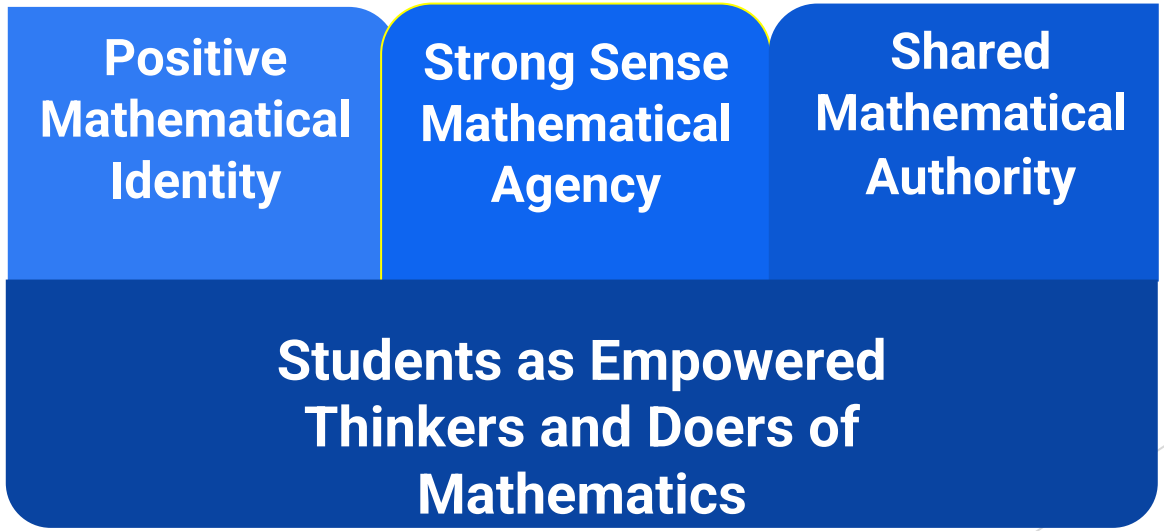
Collaborative endeavor

Share thinking

Students as Empowered Thinkers and Doers of Mathematics

Recommendation #3
Implement Equitable Mathematics Instruction

- Quality of mathematics learning experiences rather than quantity of problems
- Mathematics is seen as a collaborative endeavor
- Students are asked to solve problems in more than one way
- Students are encouraged to share their thinking, not just solutions



Mathematics Teaching Practices

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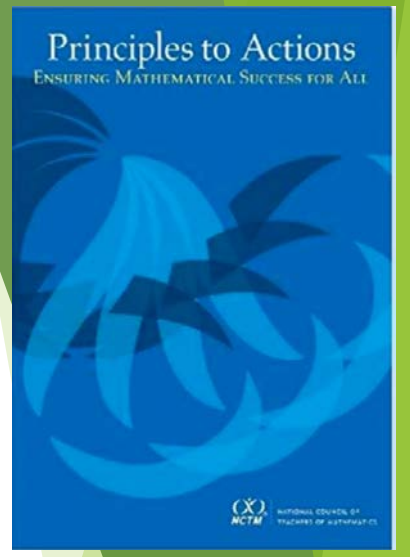
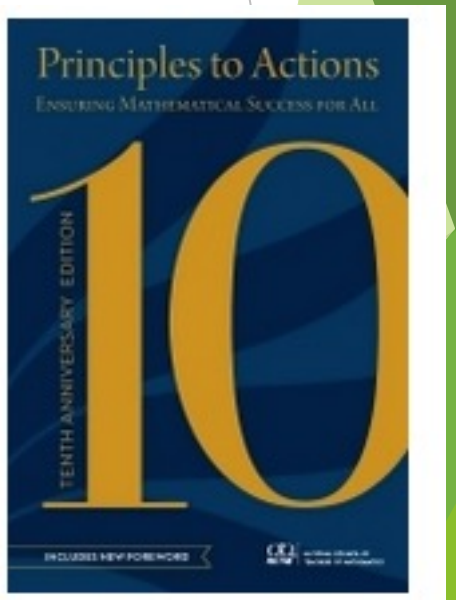
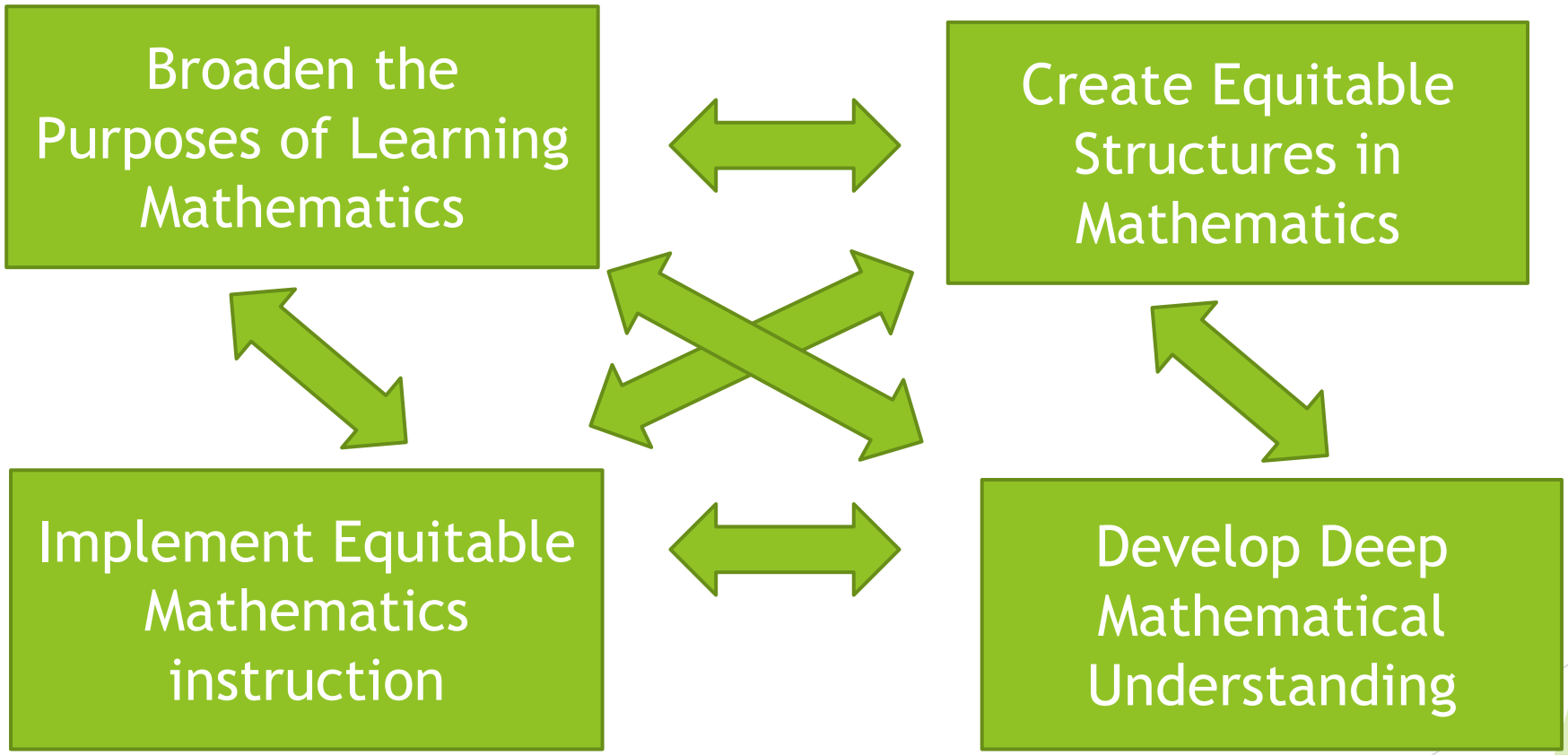




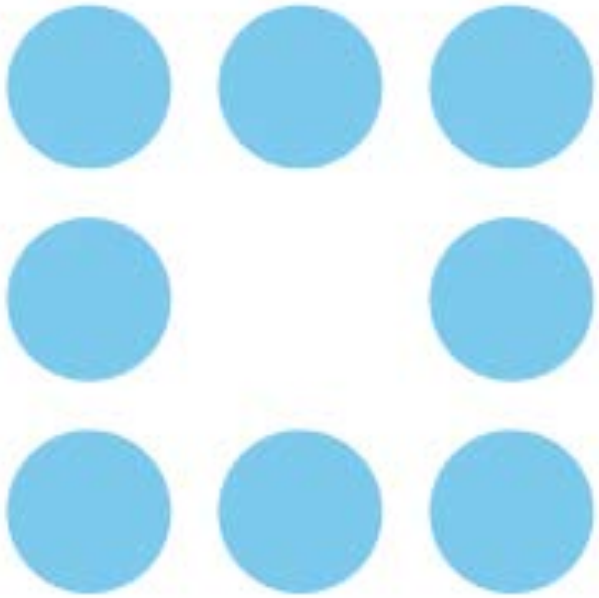
Consider Connections



Catalyzing Change Series, 2018, 2020, NCTM

3

How many dots do you see? Can you tell without counting? Explain your thinking to a friend.



MTLT, April 2020
Problems to Ponder
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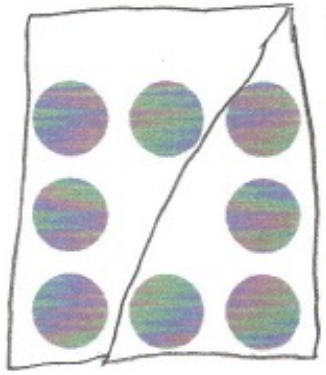
Student Work- Kinder

How many dots do you see? Can you tell without counting? Explain your thinking to a friend.

*Student stated there were 8 dots



Student writing

- Student counted dots in head

How do you know?

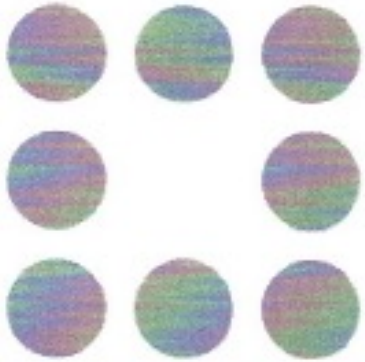
R: because 4 and 4 make 8

Can you show me where you saw the 4 and 4?

Student counted 2 sets of 4 and showed the sets which are shown by the boxes on the image

How many dots do you see? Can you tell without counting? Explain your thinking to a friend.

* student stated there were 7 dots



student writing

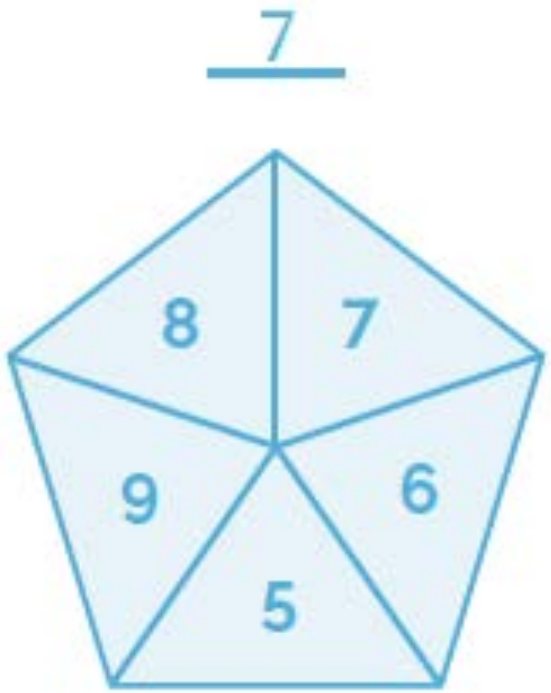
7

* noticed student counted

* How do you know

R: because my brain thinks.

Student Work



Jamila is playing a game where she throws darts at a pentagonal dart board. She can throw up to 5 darts, and her goal is to get a score of 30. What are different ways that she can reach the target score?

MTLT, May 2020
Problems to Ponder
p. 433

9+9=18

18+5=23

23+7=30

5+9=14

14+9=24

24+6=30

= 30

6+6+6+6+6=30

6+6=12 12 1

6+6=12 +12 24

6+6=12 24 +6

30

5+6=11 11

5+6=11 −11

+11

22

1 2

+ 8

30

8+8=16

16+8=24

24+6=30

6+6=12

12+8=20

20+5=25

25+5=30

5+5=10

10+6=16

16+9=25

25+5=30

9

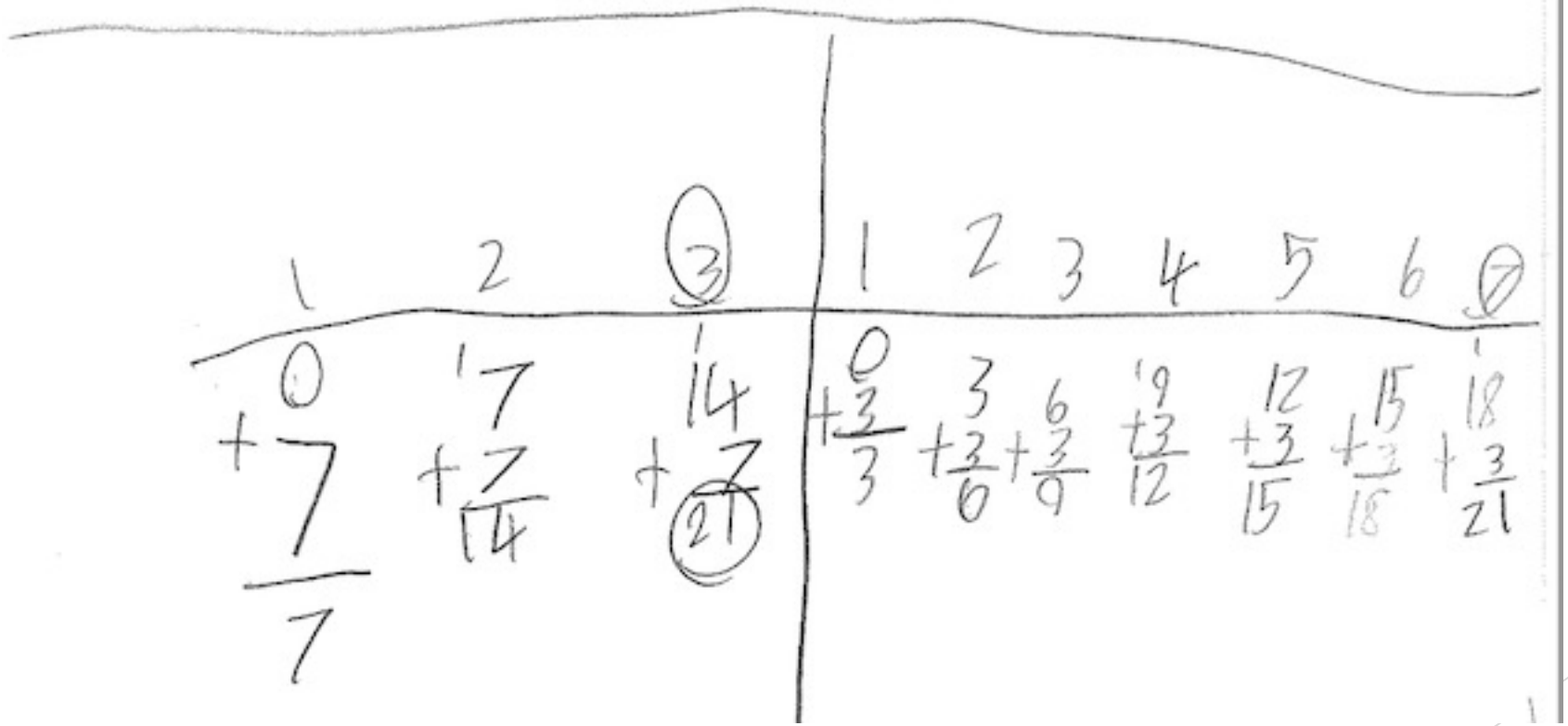
Fred says that 7×3 is the same as 3×7. Gina disagrees and says that they are different. Who do you agree or disagree with, Fred or Gina? Use a mathematical drawing to demonstrate your reasoning.

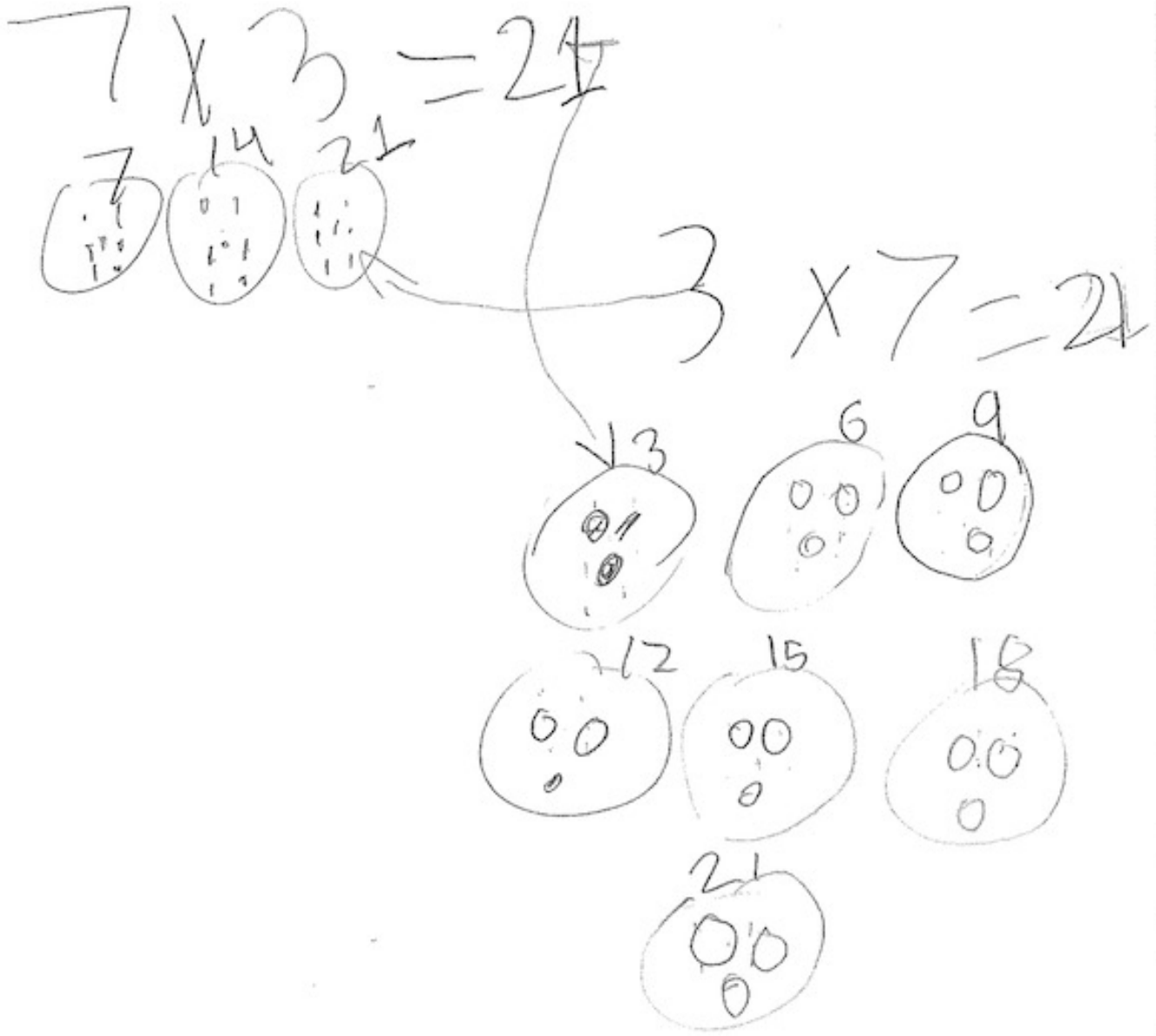
Grade 3

MTLT, June 2020
Problems to Ponder
p. 530

a mathematical drawing to demonstrate your reasoning.

I agree with Fred because why... Gina saying "7X3 is different than 3X7" It's just switching up the numbers. If you do all the work you still get the same answer.





What do you notice?
What do you Wonder?

Impact & Purpose

- Offer problems that allow for exploration, social interaction, and material engagements. (Noelle Parks, MTLT, January 2020, p. 61)
- Allow for student to play with mathematics (Denger, MTLT February 2022)
- Mathematical Discussions, gain new insights, learn from each other's strategies (Semper, 4th Grade, Castleman Creek Elementary, Texas)

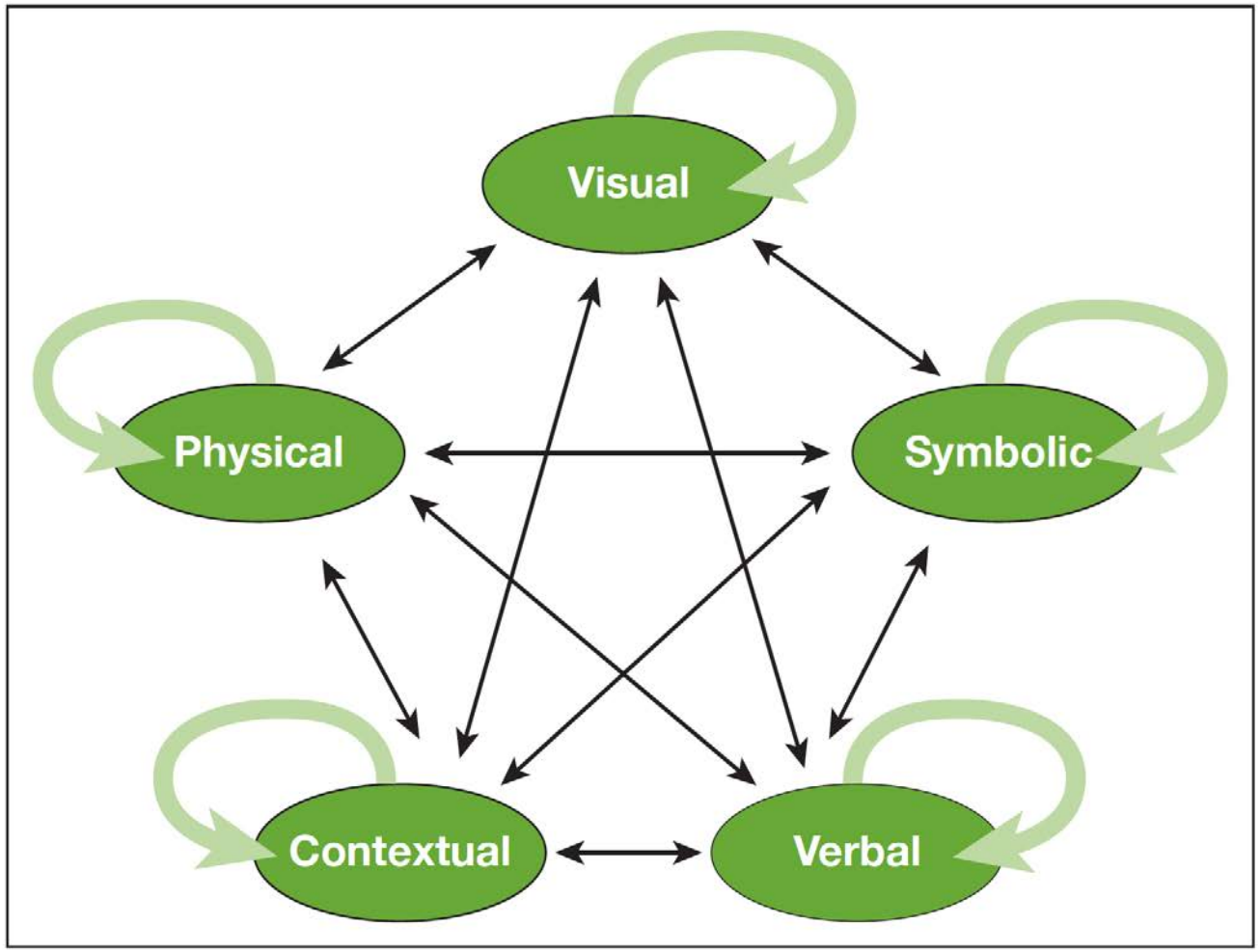


Fig. 4.6 Connections between and within mathematical representations (Huinker 2015, p. 6)

Huinker, DeAnn.2015. “Representational Competence: A Renewed Focus for Classroom Practice in Mathematics.” Wisconsin Teacher of Mathematics 67, no. 2 (Spring): 4–8.

MATHEMATICS TEACHER
Learning & Teaching
PK
12

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

PUBS.NCTM.ORG

PROBLEMS_TO_PONDER

Answer key available at nctm.org/mtlt11501p2p.

Problems to Ponder

Problems to Ponder provides 28 varying, classroom-ready mathematics problems that collectively span PK–12, arranged in the order of the grade level. Answers to the problems are available online. Individuals are encouraged to submit a problem or a collection of problems directly to mtlt@nctm.org. If published, the authors of problems will be acknowledged.

Teacher Reflection

"It's important to me that my students engage in problem-solving and problem-solving related tasks because I want to help build a generation of mathematicians that are able to think, discuss, and persevere when working through problems that can be challenging."

Guadalupe Siclla
Grade 3 Gardens ES
Pasadena ISD, TX

Why Problem Solving?

Implement tasks that promote reasoning and problem solving

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

(PtA, 2014, p. 10).

"Mathematics is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean to be doing mathematics? In the first place, it means to be able to solve mathematical problems."

Polya (from a lecture on teaching)

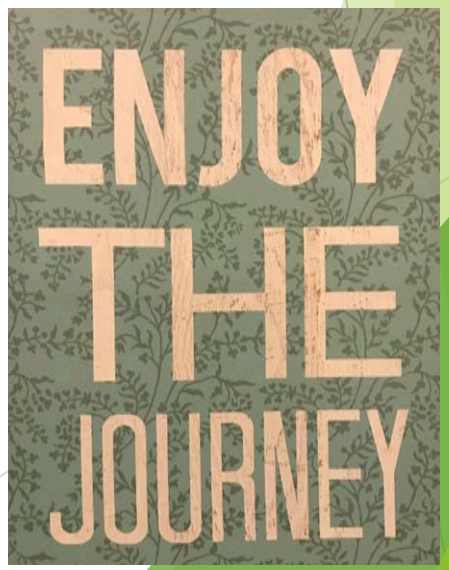
https://www.cmc-math.org/george-polya

Guiding Questions

- How does problem solving enhance creativity and support critical thinking and why is it important?
- What are effective mathematics teaching practices that enhance student thinking and engage students in problem solving?
- What mathematical practices do we want our students to develop?
- What is the role of productive struggle in teaching and learning maths?

Teaching math is a journey we take with not only our students but our colleagues, friends, family, and more each day, week, month, and year, over our lifetime……let’s do it together!

(Wilkerson, 2020)



Plan your next steps!

My Next Steps?

What will I do?

What will we do?

Thank you!
Have a wonderful conference!

Trena_Wilkerson@Baylor.edu

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- Parks, Amy Noelle. 2020. “Creating Joy in PK-Grade 2. Mathematics Classrooms.” *Mathematics Teacher: Learning & Teaching PK-12* 113, no. 1 (January): 61-64.

Other Problems

Geometry/Measurement

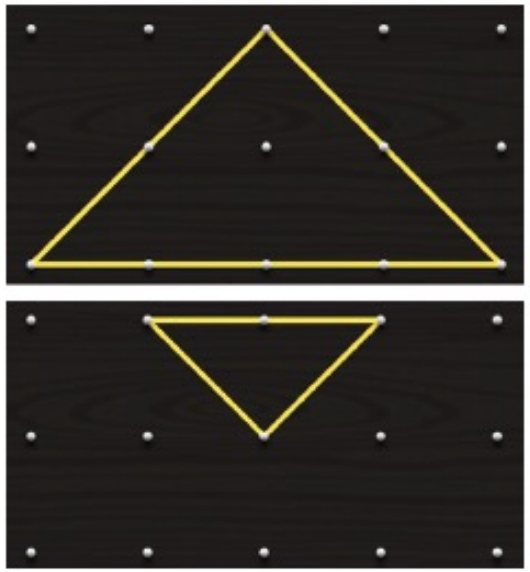
14

Lindsay drew a rectangle. She drew another rectangle that was double the length and double the width of the first rectangle. She noticed something interesting about the area of the new rectangle, so she drew a third one. For this third rectangle, she doubled the length and width of the second rectangle. She repeated this process to draw a fourth rectangle. What do you think Lindsay was noticing about the relationship between the areas of the rectangles she made? Why might this be happening?

MTLT, February 2020 Problems to Ponder p. 173

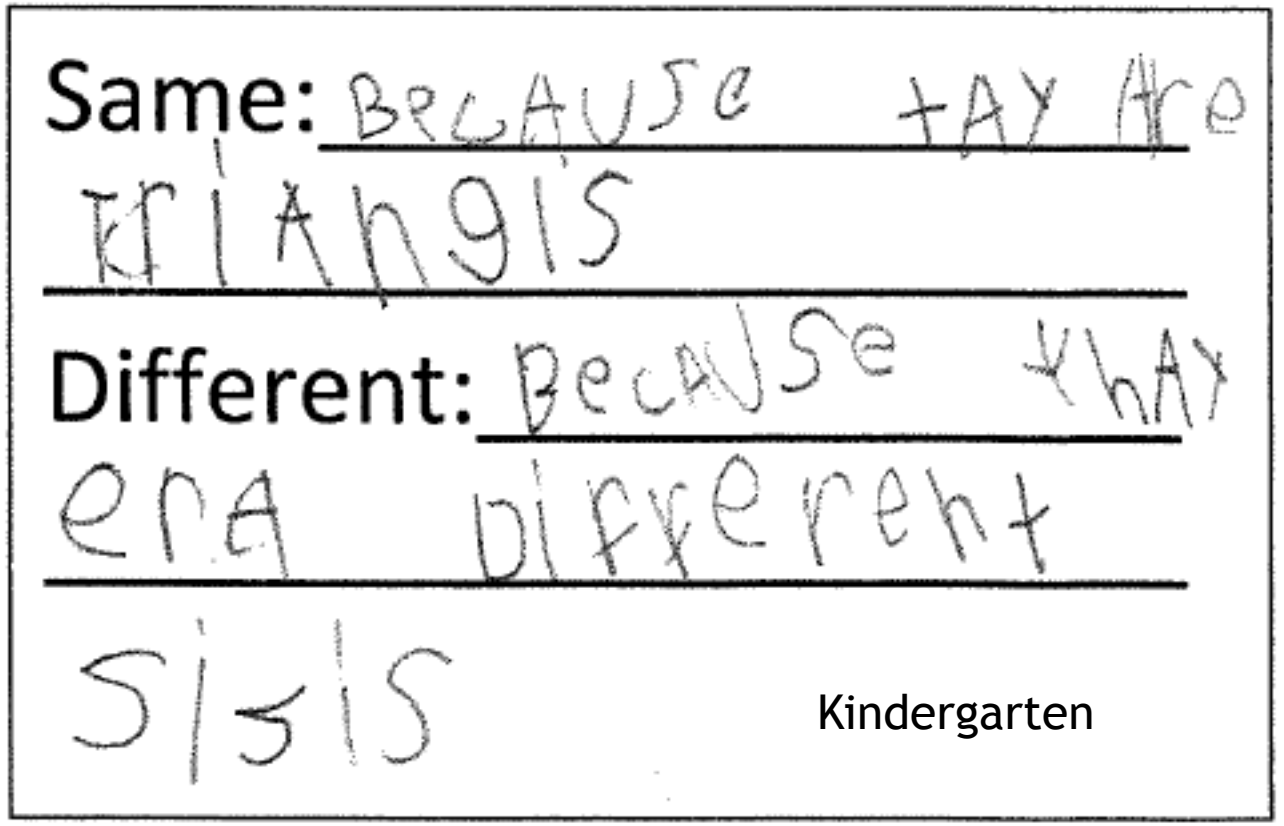
4

In these pictures, what is the same? What is different?



Use objects around you to make triangles. You could use crayons, pretzel sticks, craft sticks, paper clips, or clay.

MTLT, October 2021
Problems to Ponder
p. 788



Kindergarten

Same: BE CUSTHADP
hcRI INGGLG

Different: ONE ISSMOL
ANA ONE ISBIG

Same: TAYhaVEJ
SISS

Different: TAYARE
DIFFERENT
SISIS

Order of Operations

13

Use the digits zero to nine once each, so that each expression simplifies to a different odd number.

$$\square + \square \times \square$$

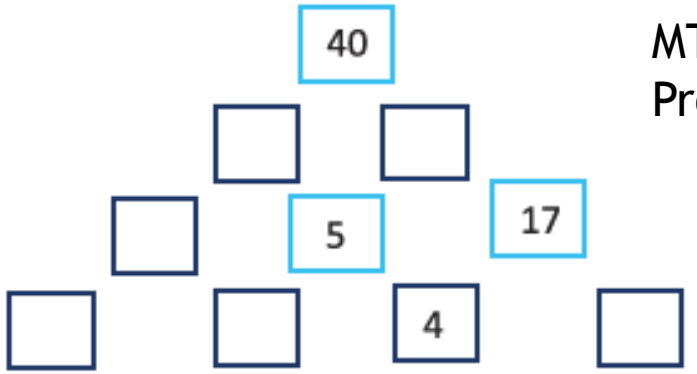
$$\square - \square \div \square$$

$$\square \div (\square - \square) \times \square$$

MTLT, December 2021 Problems to Ponder p. 971

8

Pyramid puzzle: Each number in the pyramid is the sum of the two numbers below it. Fill in the missing numbers in the pyramid. Numbers may repeat.

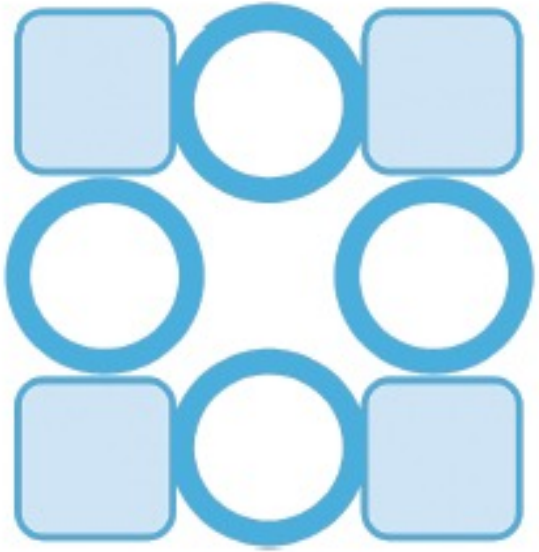


MTLT, May 2021
Problems to Ponder p. 401

8

Using 8 numbers from 1 to 10, place one number in each shape, so that each number in each circle is the sum of the two adjacent shapes.

MTLT, December 2021
Problems to Ponder p. 970



25

A farmer has chickens and cows. There are 16 heads and 38 feet among the animals on the farm. How many cows does the farmer have?

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8

Draw a number line. On one end, mark 200, and on the other end, mark 400. About where would 275 be on the number line? 368? 223? Show where you would place each number and share your reasoning.

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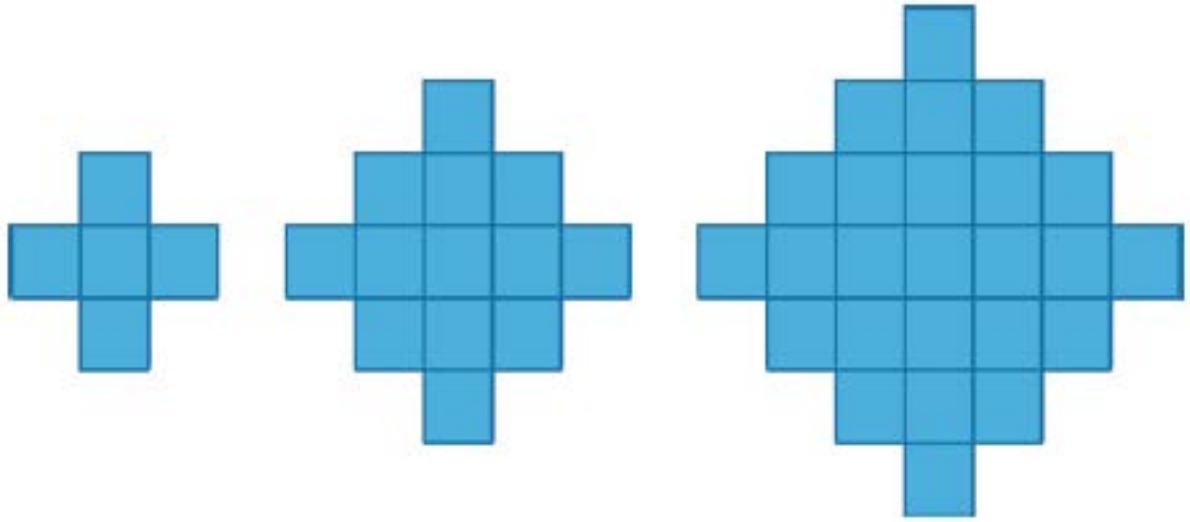
23



Write a function to describe the number of squares found in the nth term of the sequence.

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