(ON)WAARHEDEN EN (ON)BEKENDE ZAKEN UIT ONDERZOEK OVER REKEN-WISKUNDEONDERWIJS

Christian Bokhove

Panama

1 Juli 2022

Wie ik ben

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- Van 1998-2012 wiskunde docent, Zaandam, Nederland
- Nationale projecten wiskunde en technologie, Universiteit Utrecht
- Universitair hoofddocent aan de University of Southampton, United Kingdom
  - Wiskunde-onderwijs
  - Technologie
  - Large-scale assessment (PISA/TIMSS)
  - Onderzoeksmethoden
- In Engeland basisschool tot 11 jaar, gemeenschappelijke thema’s, website gericht op basisscholen.

Het valt niet mee om wijs te worden uit onderzoek dat relevant is voor het reken- en wiskundeonderwijs.

Gaat het goed of niet?

“Onderzoek wijst uit….” – alles lijkt wel te bewijzen.

“Het rekenonderwijs is kapot gemaakt door….”

Vaak ligt het helemaal niet zo simpel.

Deze lezing

In deze lezing kijk ik naar verschillende bronnen die worden aangehaald bij discussies over het reken-wiskundeonderwijs.

- Wat internationale en nationale peilingen over de stand van ons reken-wiskundeonderwijs zeggen.
- Het complementaire karakter van vaardigheden en inzicht.
- Onderzoeksthema’s die veelvuldig langskomen in discussies, bijvoorbeeld met de inspectie en de recente NPO menukaart. Van enkele van deze thema’s, geef ik concrete voorbeelden hoe onderzoek soms ge- en mis-bruikt wordt.

INTERNATIONALE EN NATIONALE PEILINGEN

Wat kunnen de peilingen zeggen?

- Veel beperkingen!
- Data verzameld zonder onderzoeksvraag
- Secundaire data analyse
- Maar: enige manier grootschalige, internationale vergelijkingen.
- Beperkingen
  - Kan niet zomaar analyseren: complex sampling design: gewichten, plausible values, resampling
  - Correlatie/causatie

Opinion

Memorizers are the lowest achievers and other Common Core math surprises

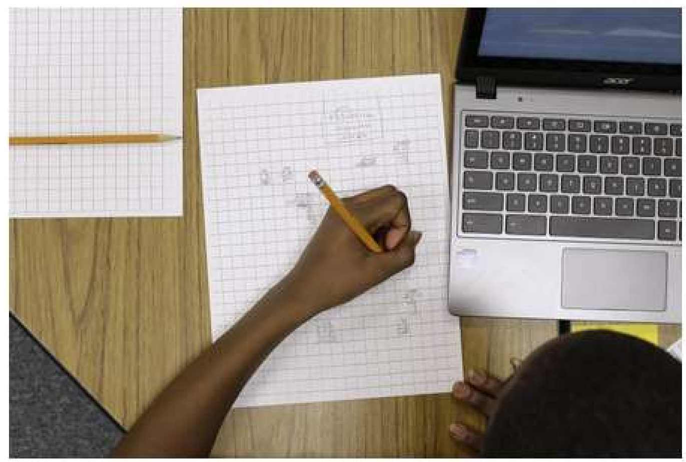
by JO BOALER

May 7, 2015

It's time to debunk the myths about who is good in math, and Common Core state standards move us toward this worthy goal. Mathematics and technology leaders support the standards because they are rooted in the new brain and learning sciences.

All children are different in their thinking, strength and interests. Mathematics classes of the past decade have valued one type of math learner, one who can memorize well and calculate fast.

Yet data from the 13 million students who took PISA tests showed that the lowest achieving students worldwide were



In this Feb. 12, 2015 photo, Yamarko Brown, age 12, works on math problems as part of a trial run of a new state assessment test at Annapolis

esproken
aan
. Het is
en, maar

derwijs dat
op het
gsten van al
e ranglijsten
et zelfs
a in het

Vietnam 17
France 25

Bron: https://www.groene.nl/artikel/futuristische-retoriek-of-evidence-based (8 Mei 2019)

Bron: http://hechingerreport.org/memorizers-are-the-lowest-achievers-and-other-common-core-math-surprises/ (7 Mei 2015)

Bron: http://www.bbc.co.uk/news/education-25187997 (3 December 2013)

OECD	Mean score in PISA 2018			Long-term trend: Average rate of change in performance, per three-year-period			Short-term change in performance (PISA 2015 to PISA 2018)			Top-performing and low-achieving students	
	Reading	Mathematics	Science	Reading	Mathematics	Science	Reading	Mathematics	Science	Share of top performers in at least one subject (Level 5 or 6)	Share of low achievers in all three subjects (below Level 2)
	Mean	Mean	Mean	Score dif.	Score dif.	Score dif.	Score dif.	Score dif.	Score dif.	%	%
OECD average	487	489	489	0	-1	-2	-3	2	-2	15.7	13.4
Estonia	523	523	530	**6**	**2**	0	4	4	-4	22.5	4.2
Canada	520	512	518	-2	**-4**	**-3**	-7	-4	**-10**	24.1	6.4
Finland	520	507	522	**-5**	**-9**	**-11**	-6	-4	**-9**	21.0	7.0
Ireland	518	500	496	0	0	**-3**	-3	-4	-6	15.4	7.5
Korea	514	526	519	**-3**	**-4**	**-3**	-3	2	3	26.6	7.5
Poland	512	516	511	**5**	**5**	2	6	**11**	**10**	21.2	6.7
Sweden	506	502	499	**-3**	-2	-1	6	8	6	19.4	10.5
New Zealand	506	494	508	**-4**	**-7**	**-6**	-4	-1	-5	20.2	10.9
United States	505	478	502	0	-1	2	8	9	6	17.1	12.6
United Kingdom	504	502	505	2	1	-2	6	**9**	-5	19.4	9.0
Japan	504	527	529	1	0	-1	**-12**	-5	**-9**	23.3	6.4
Australia	503	491	503	**-4**	**-7**	**-7**	0	-3	**-7**	18.9	11.2
Denmark	501	509	493	1	-1	0	1	-2	**-9**	15.8	8.1
Norway	499	501	490	1	2	1	**-14**	-1	**-8**	17.8	11.3
Germany	498	500	503	**3**	0	**-4**	-11	-6	-6	19.1	12.8
Slovenia	495	509	507	2	2	**-2**	**-10**	-1	**-6**	17.3	8.0
Belgium	493	508	499	-2	**-4**	**-3**	-6	1	-3	19.4	12.5
France	493	495	493	0	**-3**	-1	-7	2	-2	15.9	12.5
Portugal	492	492	492	**4**	**6**	**4**	-6	1	**-9**	15.2	12.6
Czech Republic	490	499	497	0	**-4**	**-4**	-3	7	-4	16.6	10.5
Netherlands	485	519	503	**-4**	**-4**	**-6**	**-18**	7	-5	21.8	10.8
Austria	484	499	490	-1	-2	**-6**	0	2	-5	15.7	13.5

Bron: https://www.oecd.org/pisa/Combined_Executive_Summaries_PISA_2018.pdf

Maar nu ook tegengas

- Veel uitspraken over waar het door komt, zijn nogal kort door de bocht.
- Helaas, alleen PISA 2012 had wiskunde-focus dus we hebben geen extra informatie over zaken zoals memorisering.
- Al valt over die vragen toch ook veel te zeggen.

Q41 For each group of three items, please choose the item that best describes your approach to mathematics.

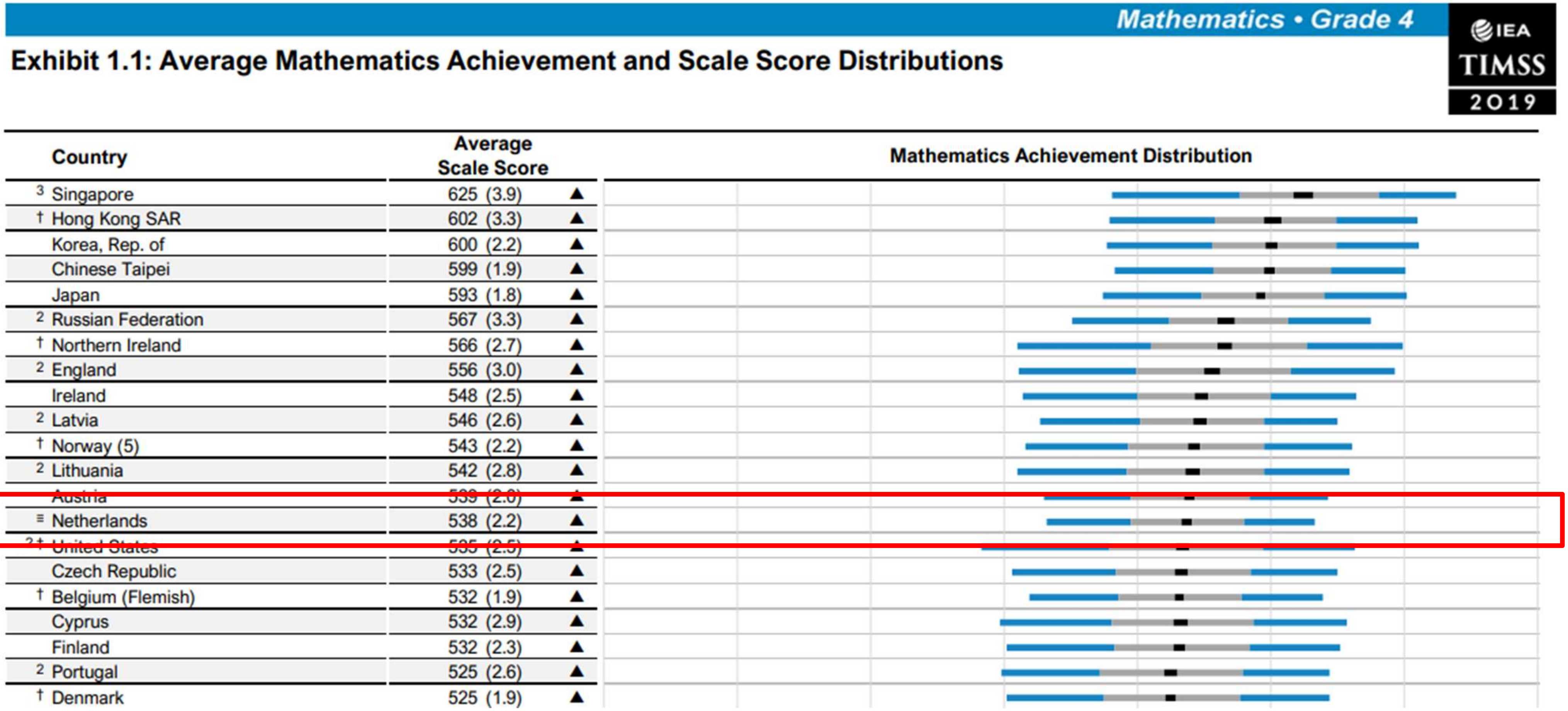
a) Please tick only one of the following three boxes.

☐ 1 When I study for a mathematics test, I try to work out what the most important parts to learn are.

☐ 2 When I study for a mathematics test, I try to understand new concepts by relating them to things I already know.

☐ 3 When I study for a mathematics test, I learn as much as I can off by heart.

- Mijn advies: elke bron is startpunt niet eindpunt.



Mathematics • Grade 4

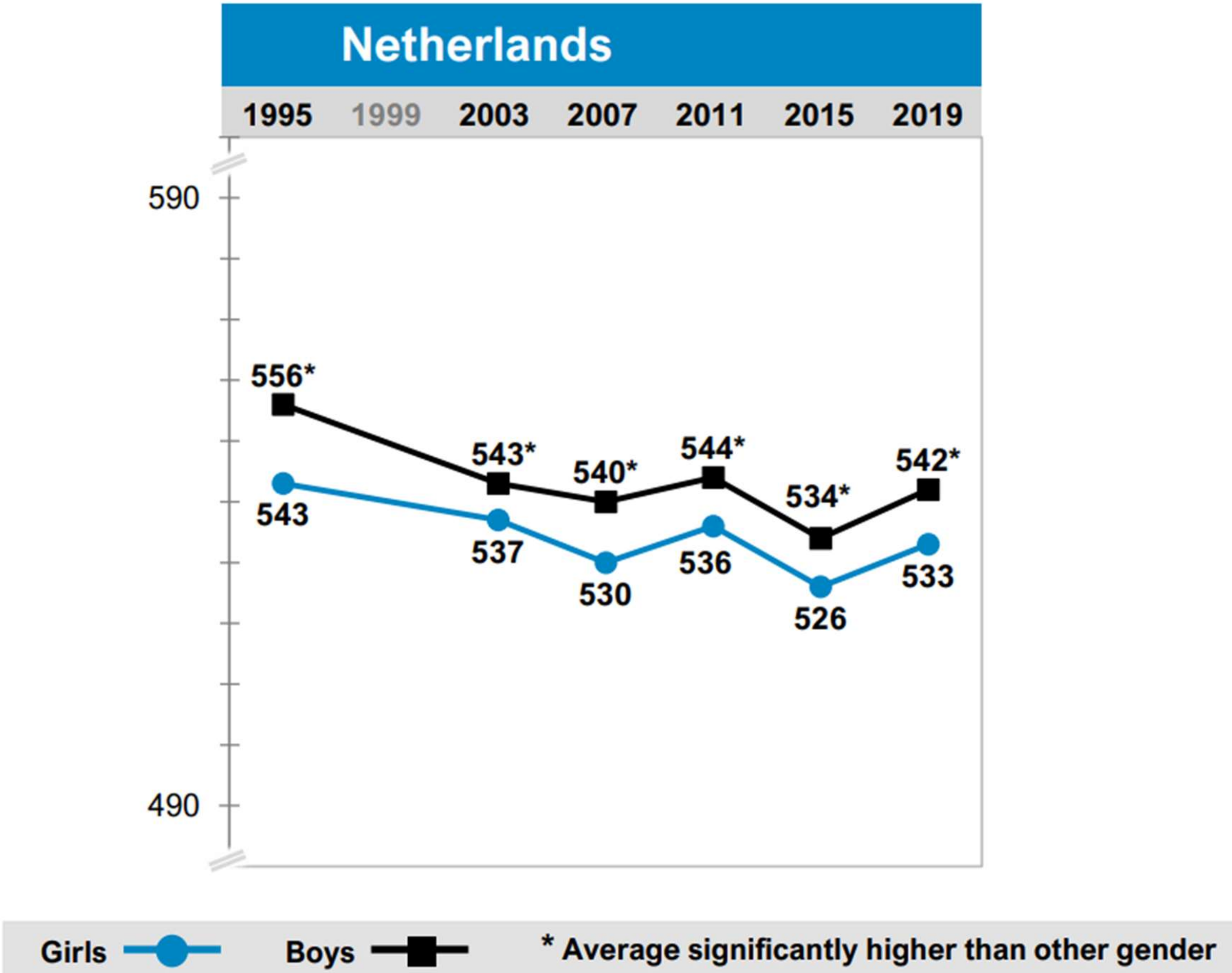
Exhibit 1.1: Average Mathematics Achievement and Scale Score Distributions

IEA TIMSS 2019

Country	Average Scale Score		Mathematics Achievement Distribution
3 Singapore	625 (3.9)	▲	
† Hong Kong SAR	602 (3.3)	▲	
Korea, Rep. of	600 (2.2)	▲	
Chinese Taipei	599 (1.9)	▲	
Japan	593 (1.8)	▲	
2 Russian Federation	567 (3.3)	▲	
† Northern Ireland	566 (2.7)	▲	
2 England	556 (3.0)	▲	
Ireland	548 (2.5)	▲	
2 Latvia	546 (2.6)	▲	
† Norway (5)	543 (2.2)	▲	
2 Lithuania	542 (2.8)	▲	
Austria	539 (2.0)	▲	
≡ Netherlands	538 (2.2)	▲	
2 † United States	535 (2.5)	▲	
Czech Republic	533 (2.5)	▲	
† Belgium (Flemish)	532 (1.9)	▲	
Cyprus	532 (2.9)	▲	
Finland	532 (2.3)	▲	
2 Portugal	525 (2.6)	▲	
† Denmark	525 (1.9)	▲	

≡ Did not satisfy guidelines for sample participation rates

Bron: https://timss2019.org/reports/achievement/#math-4


Netherlands
1995
1999
2003
2007
2011
2015
2019
590
490
556*
543*
540*
544*
534*
542*
543
537
530
536
526
533
Girls
Boys
* Average significantly higher than other gender

- Heel laag ‘advanced’ (stijgend) maar hoog ‘low benchmark’ (daalde, maar 98%)
- Content domains: ‘Data’ hoger tov 2015 maar lager 2011, ‘Number’ hetzelfde tov 205, lager dan daarvoor, ‘Measurement and Geometry’ verbeterd (maar was voorheen lager).
- Cognitive domains: ‘Knowing’ flink gestegen tov 2015, ‘Applying’ iets beter, bijna niveau, ‘Reasoning’ iets hoger.
- Maar let wel: versterken elkaar.

Maar wat wordt eigenlijk gemeten?

- PISA meer mathematical *literacy*
- Allebei zeer bepaald door IQ, maar PISA meer dan TIMSS (Rindermann, 2007).
- TIMSS meer curriculum georienteerd. (Rindermann & Baumeister, 2015)
- TIMSS-PIRLS relationship report: slechtere lezers deden het slechter dan betere lezers maar extra benadeeld op vragen met meer leeswerk (Mullis et al., 2013).

En dan ook…

Tabel 3.1 PPON effectgroottes en vergelijking met standaarden

Onderwerpen in PPON-2004 einde basisonderwijs	Effectgrootte verschil in peil 2004-1987			Percentage leerlingen dat de standaard 'voldoende' haalt		
	< – 0,19		> + 0,19	< 40%	40-60%	> 60%
GETALLEN EN BEWERKINGEN						
1. getallen en getalrelaties			+ 0,94		42	
2. basisoperaties: + / –			+ 0,24			76
3. basisoperaties: x / :	– 0,20					66
4. hoofdrekenen: + / –			+ 0,53		50	
5. hoofdrekenen: x / :		– 0,11				66
6. schattend rekenen			+ 1,04		42	
7. bewerkingen: + / –	– 0,53			27		
8. bewerkingen: x / :	– 1,16			12		
9. samengestelde bewerkingen	– 0,78			16		
10. rekenen met een zakrekenmachine			+ 0,26	34		
VERHOUDINGEN, BREUKEN EN PROCENTEN						
11. verhoudingen		+ 0,14				66
12. breuken		+ 0,15				60
13. procenten			+ 0,51		58	
14. tabellen en grafieken		+ 0,10			50	
METEN EN MEETKUNDE						
15. meten: lengte		– 0,13		38		
16. meten: oppervlakte		+ 0,05		21		
17. meten: inhoud		– 0,03			42	
18. meten: gewicht			+ 0,33		58	
19. meten: toepassingen	– 0,25				50	
20. meetkunde		– 0,08				62
21. tijd		0,00			50	
22. geld	– 0,31				42	

De effectgroottes betreffen periode 2004-1987 met uitzondering van de effectgroottes voor onderwerpen *2+3 basisoperaties* en *10 rekenen met een zakrekenmachine* (2004-1992); *14 tabellen en grafieken* (2004-1997); *22 geld* (1997-1987).
Bron: Van der Schoot (2008).

PPON 2004

Terug naar die koppen….

Kortom, de claims zijn het **STARTPUNT**, niet het **EINDPUNT**

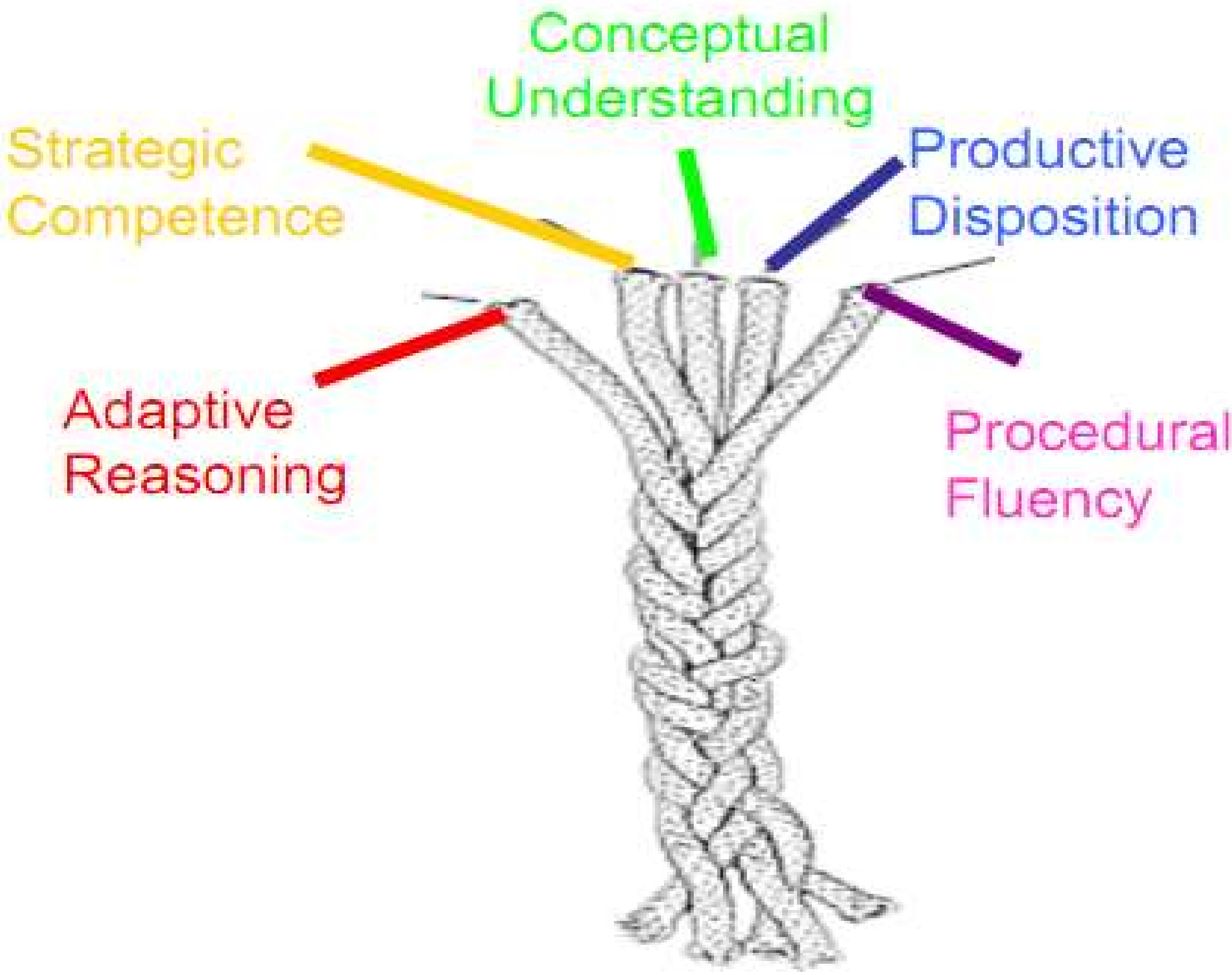
COMPLEMENTAIRE KARAKTER VAN VAARDIGHEDEN EN INZICHT

The Math Wars

ALAN H. SCHOENFELD

During the 1990s, the teaching of mathematics became the subject of heated controversies known as the math wars. The immediate origins of the conflicts can be traced to the "reform" stimulated by the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics. *Traditionalists fear that reform-oriented, "standards-based" curricula are superficial and undermine classical mathematical values; reformers claim that such curricula reflect a deeper, richer view of mathematics than the traditional curriculum. An historical perspective reveals that the underlying issues being contested—Is mathematics for the elite or for the masses? Are there tensions between "excellence" and "equity"? Should mathematics be seen as a democratizing force or as a vehicle for maintaining the status quo?—are more than a century old. This article describes the context and history, provides details on the current state, and offers suggestions regarding ways to find a productive middle ground.*

Schoenfeld, A. H. (2004). The math wars. *Educational Policy*, *18*(1), 253–286.



(Kilpatrick et al., 2001)

Educ Psychol Rev
DOI 10.1007/s10648-015-9302-x

REVIEW ARTICLE

Not a One-Way Street: Bidirectional Relations Between Procedural and Conceptual Knowledge of Mathematics

Bethany Rittle-Johnson · Michael Schneider · Jon R. Star

Abstract There is a long-standing and ongoing debate about the relations between conceptual and procedural knowledge (i.e., knowledge of concepts and procedures). Although there is broad consensus that conceptual knowledge supports procedural knowledge, there is controversy over whether procedural knowledge supports conceptual knowledge and how instruction on the two types of knowledge should be sequenced. A review of the empirical evidence for mathematics learning indicates that procedural knowledge supports conceptual knowledge, as well as vice versa, and thus that the relations between the two types of knowledge are bidirectional. However, alternative orderings of instruction on concepts and procedures have rarely been compared, with limited empirical support for one ordering of instruction over another. We consider possible reasons for why mathematics education researchers often believe that a conceptual-to-procedural ordering of instruction is optimal and why so little research has evaluated this claim. Future empirical research on the effectiveness of different ways to sequence instruction on concepts and procedures is greatly needed.

Siegler: wat voorspelt wiskundeprestatie?

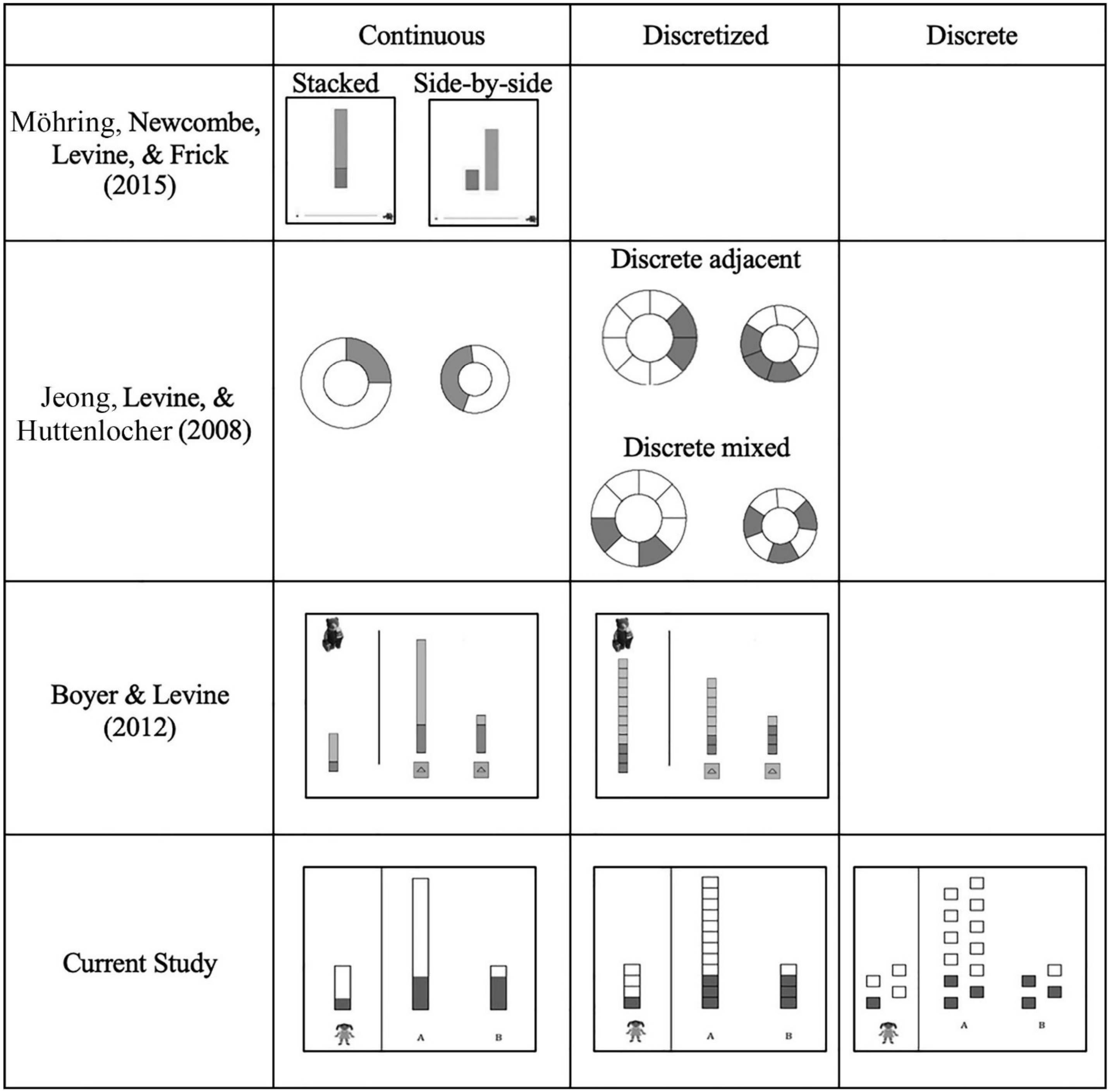
"Analyses of large, nationally representative, longitudinal data sets from the United States and the United Kingdom revealed that elementary school students' **knowledge of fractions and of division** uniquely predicts those students' knowledge of algebra and overall mathematics achievement in high school, 5 or 6 years later, even after statistically controlling for other types of mathematical knowledge, general intellectual ability, working memory, and family income and education."

Siegler et al. (2012)

Onderzoek getallen en operaties

- Grootte van getallen
  - Grotere getallen=moeilijker
  - ‘Mooie getallen’ en compensatiestrategie

- Rol van ‘gelijk’ teken (bv McNeil & Alibali, 2005, McNeil et al., 2019)
  - Operationeel
  - Relationeel
  - Substitutie



Begolli net al. (2020) met 565 7- tot 12-jarigen

Kortom, de claims zijn het **STARTPUNT**, niet het **EINDPUNT**

ENKELE RELEVANTE THEMA'S

THEMA: TAAL

"…the present findings provide a more detailed and deeper insight into the learning process of children's number concept, emphasizing the influence of phonological and particularly grammatical skills."

Taal heeft behoorlijke invloed.

PLOS ONE

RESEARCH ARTICLE

How language skills and working memory capacities explain mathematical learning from preschool to primary school age: Insights from a longitudinal study

Nurit Viesel-Nordmeyer[1,2]*, Alexander Röhm[2], Anja Starke[3], Ute Ritterfeld[2]

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Check for updates

OPEN ACCESS

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Competing interests: The authors have declared that no competing interests exist.

Abstract

Between the age span of 3 to 6 years the foundation for children's mathematical learning (i.e., numerical abilities and cognition) are laid. However, the developing relations between mathematical skills, language, and working memory starting at preschool age and evolving into primary school age are not well understood. Adopting an empirically validated analysis model, the present study examines in detail longitudinal interdependencies between mathematical skills, a wide range of language skills, and working memory components underlying the mathematical learning process of 41 German preschool children (41.5% female) spanning ages 4 to 6. Phonological processing skills and expressive grammar skills emerged as the most significant language skills for the process of children's mathematical learning across the investigated age span. Within the latter, children's phonological processing skills and expressive grammar skills were supported by children's word expression abilities. The phonological loop emerged as the most important working memory component for children's early mathematical learning between ages 4 to 6. Furthermore, a wide array of language skills were associated with complex information and storage processes within this mathematical learning process. In conclusion, the present findings provide a more detailed and deeper insight into the learning process of children's number concept, emphasizing the influence of phonological and particularly grammatical skills.

Introduction

Understanding the process of early mathematical learning has been the aim of many scholars over the past decades. During recent years, it has become increasingly obvious that children's mathematical skills are often linked to their language development (e.g., [1, 2]) as well as their working memory capacity (e.g., [3, 4]), or a combination of both (e.g., [5–8]). However, the longitudinal relations between all three components—language skills, working memory and mathematical skills—underlying this process of mathematical learning from preschool to

Other examples

Arcavi (1994): "The findings in Clement [1982] show that more than 30% of the 150 freshman engineering students who answered the test failed to solved this problem correctly. The typical wrong answer reported was *6S = P.*"

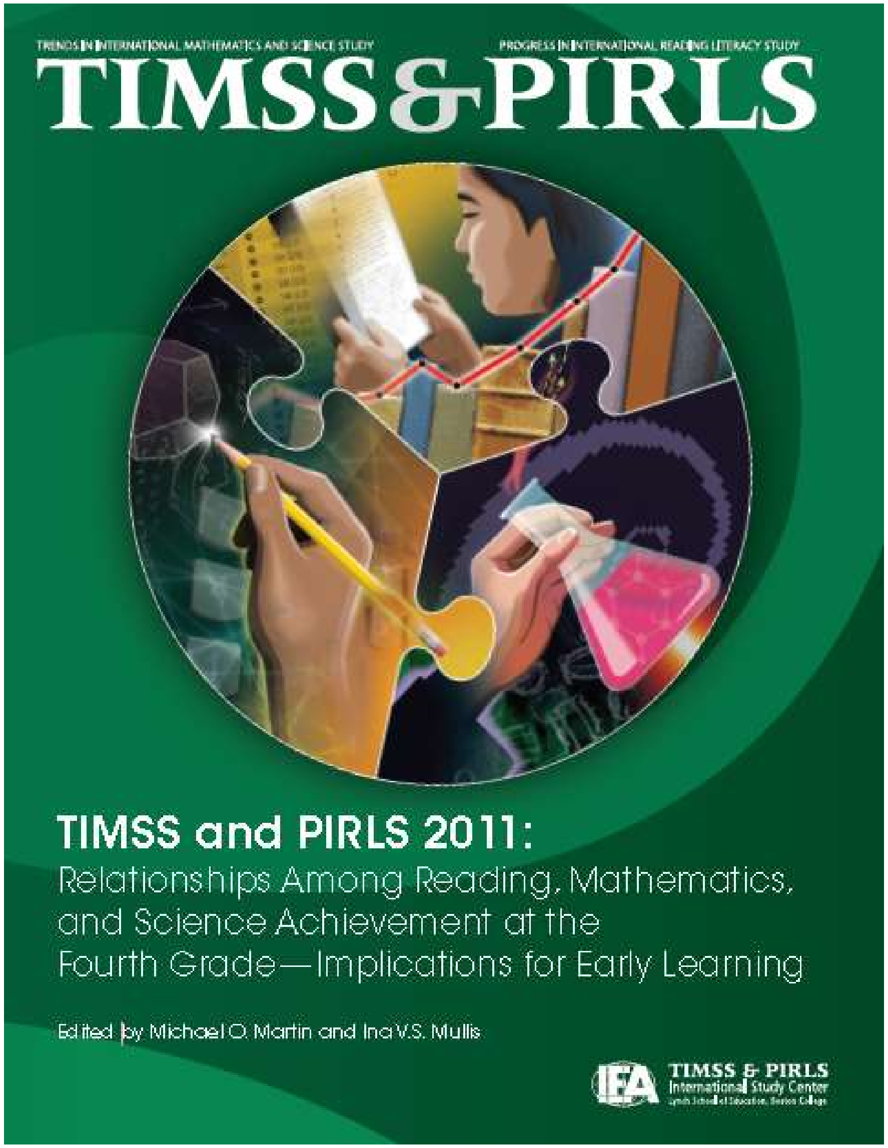
> "Write an equation using the variables S and P to represent the following statement: There are six times as many students as professors at this university." [See, for example, Clement, 1982]

Hij merkte op dat er een afleider was:

"language distractor, i e. the order of the key words (six times as many students) which may bias our sense towards a "word-by-word translation" as *6S = P*"

TIMSS-PIRLS relatie report

- In 2011 dezelfde basisschool-leerlingen TIMSS en PIRLS (lees-equivalent TIMSS).
- Bestuderen relatie lezen en wiskunde.
  - Landprofielen
  - Het effect van leesvaardigheid
  - Effectieve scholen
  - Invloed thuiscontext



https://timssandpirls.bc.edu/timsspirls2011/international-database.html

Het effect van leesvaardigheid

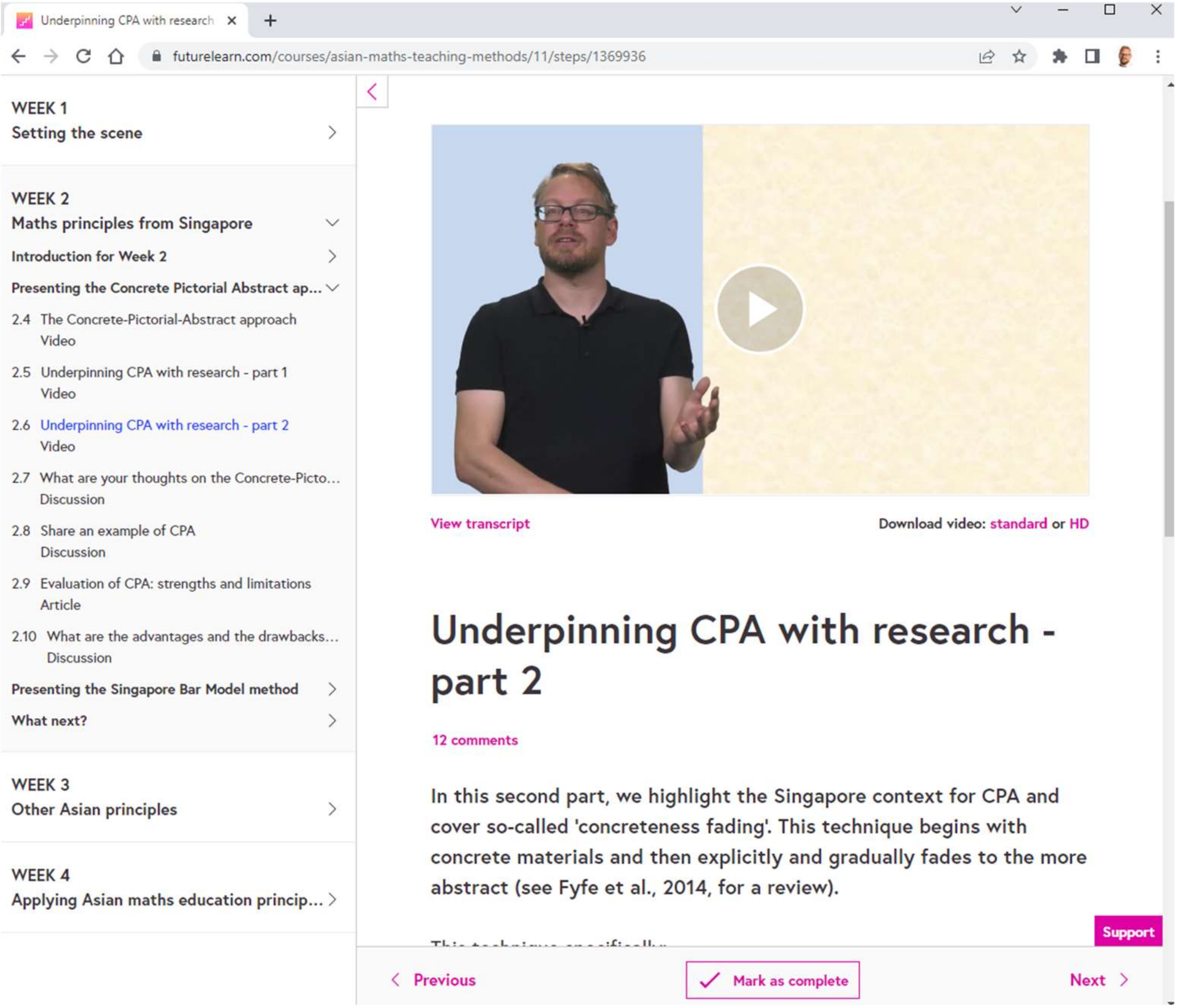
‘Reading demand’ – de leesvraag - indicatoren

- Aantal woorden.
- Symbolische taal. Hiertoe rekenen we ook getallen (e.g., 3, 5, 40) en andere symbolen en afkortingen (bijv., +, =, cm).”
- Vocabulaire. Het gebruik van bepaalde reken- of wiskunde-eigen woorden dragen bij.
- Visualisaties. Hierbij horen: 1) Grafisch representaties van bestaande objecten, 2) meetkundige vormen en figuren, 3) modellen en diagrammen, 4) tabellen, en 5) grafieken.

Bevindingen

- Gemiddeld gesproken, was het verschil in wiskundeprestatie tussen minder goede en goede lezers hoger op de vragen met een hoge 'leesvraag' dan die met een lagere 'leesvraag'.
- Howel de minst goede lezers minder goed presteerden in wiskunde dan de beste lezers, warden ze extra benadeeld op de wiskundevragen die meer leeswerk hadden.
- In de meeste landen, hadden de betere lezers voordeel over minder goede lezers, bij vragen met een hoge leesvraag.

THEMA: “IN ANDERE LANDEN…”



https://www.futurelearn.com/courses/asian-maths-teaching-methods

Raden waarom landen het goed doen

- We hebben soms een stereotype beeld van hoe andere landen zijn.
- ‘Paradox of the Asian learner’ (Biggs 1994, 1998).
- Die doen alleen aan klassikaal en veel oefenen.

- Volgens Leung (2001) zijn er echter ook culturele tegenstellingen: “product versus process; rote learning versus meaningful learning; studying hard versus pleasurable learning; extrinsic versus intrinsic motivations; whole class teaching versus individualised learning; and competence of teachers: subject matter versus pedagogy”

Memorisation and understanding

- Dahlin en Watkins (2000): memorisatie en begrip kunnen worden verbonden door betekenisvolle herhaling.

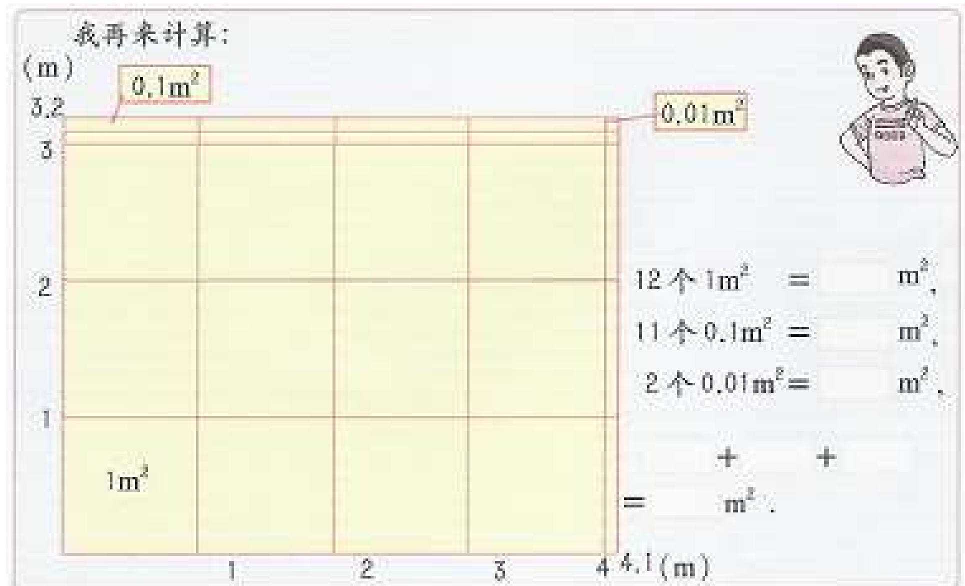
- Variation Theory bijv 'procedural variation', 1 probleem, maar verschillende methodes.

$$4.1 \times 3.2 = \boxed{13.12}$$

$$\downarrow \times 10 \quad \downarrow \times 10 \quad \uparrow \div 100$$

$$41 \times 32 = \boxed{1312}$$

$$\begin{aligned} & 4.1 \times 3.2 \\ & \boxed{= 41 \times 32 \div 100} \\ & = 1312 \div 100 \\ & = 13.12 \end{aligned}$$



THEMA: MENUKAART - INSTRUCTIE

Van april tot juni formuleren schoolleiding en leraren, in gesprek met bestuur, gemeente, ouders en anderen, de doelen voor het programma voor komend schooljaar. Ze kiezen uit de menukaart interventies die het best bij de situatie passen; die kunnen anders zijn dan het jaar ervoor.

- Menukaart gebruiken
- Opbouw menukaart
- Achtergrond menukaart

De interventies hieronder kunt u met geld uit het NP Onderwijs inzetten in het funderend onderwijs. De linkjes verwijzen naar Onderwijskennis.nl.

Soort interventie	Effect	Bewijs	Kosten

Directe instructie

- Menukaart heft het over het DI-model.
- Maar geeft ook alle verschillende verschijningsvormen aan:
  - Directe Instructie (distar)
  - directe instructie
  - Activerende Directe Instructie
  - Expliciete Directe Instructie
  - Interactieve, Gedifferentieerde, Directe Instructie
- Net als in de menukaart ‘metacognitie’ en ‘feedback’ zijn dit containertermen die lang niet altijd zo eenduidig zijn.

Wat is het dan?

- *“Five overlapping meanings of direct instruction.*

  1. Instructional procedures used in the Distar (Direct Instruction Systems in Arithmetic and Reading) programs.
  2. Academic instruction that is led by a teacher regardless of the quality of instruction.
  3. The instructional procedures that were used by effective teachers in the teacher effects research.
  4. Instructional procedures used by teachers when they taught cognitive strategies to students.
  5. Instruction where direct instruction is portrayed in negative terms such as settings where the teacher lectures and the students sit passively.”

http://www.centerii.org/search/Resources/FiveDirectInstruct.pdf

What we can take from this…

Rosenshine: “Guided practice, active student participation, and fading of teacher-directed activities appear in all three meanings. Scaffolds, modelling by the teacher, and coaching of students also appear in all three.”

Maar “rather than a ‘whole system’ it should best be seen as a **set of principles derived from various sources, within which decisions ultimately have to be left to a teacher’s professional judgement.**“ (Bokhove & Campbell, 2020)

THEMA: MOTIVATIE

Social media komt het regelmatig voor dat 'motivatie' als onbelangrijk wordt neergezet
De stelling is dat motivatie voorkomt uit prestatie

- Garon-Carrier, G., Boivin, M., Guay, F., Kovas, Y., Dionne, G., Lemelin, J. P., ... & Tremblay, R. E. (2016). Intrinsic motivation and achievement in mathematics in elementary school: A longitudinal investigation of their association. *Child development*, *87*(1), 165-175.
- Ma, X., & Xu, J. (2004). The causal ordering of mathematics anxiety and mathematics achievement: a longitudinal panel analysis. *Journal of adolescence*, *27*(2), 165-179.

En inderdaad, die geven de andere richting, maar dat doet niet andere bronnen ongedaan! De vraag 'wat klopt er niet aan die bronnen' is onterecht. Het zijn goede studies. Maar is het een 'hoofdzonde' om te zeggen dat motivatie leidt tot betere prestaties? Nee!

Daarvoor moet je echt breder lezen.

- Terminologie verschilt
  - Motivatie
  - Self-concept
  - Emotions
  - Enjoyment
  - Boredom
- “positive emotions (enjoyment, pride) positively predicted subsequent achievement (math end-of-the-year grades and test scores)” en vice versa.

CHILD DEVELOPMENT

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Achievement Emotions and Academic Performance: Longitudinal Models of Reciprocal Effects

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A reciprocal effects model linking emotion and achievement over time is proposed. The model was tested using five annual waves of the Project for the Analysis of Learning and Achievement in Mathematics (PALMA) longitudinal study, which investigated adolescents' development in mathematics (Grades 5–9; *N* = 3,425 German students; mean starting age = 11.7 years; representative sample). Structural equation modeling showed that positive emotions (enjoyment, pride) positively predicted subsequent achievement (math end-of-the-year grades and test scores), and that achievement positively predicted these emotions, controlling for students' gender, intelligence, and family socioeconomic status. Negative emotions (anger, anxiety, shame, boredom, hopelessness) negatively predicted achievement, and achievement negatively predicted these emotions. The findings were robust across waves, achievement indicators, and school tracks, highlighting the importance of emotions for students' achievement and of achievement for the development of emotions.

Research has shown that children's and adolescents' emotions are linked to their academic achievement. Typically, positive emotions such as enjoyment of learning show positive links with achievement, and negative emotions such as test anxiety show negative links (for overviews, see Goetz & Hall, 2013; Pekrun & Linnenbrink-Garcia, 2014; Zeidner, 1998). However, most of the available studies were correlational and do not allow any inferences about the causal ordering of emotion and achievement over time. As such, it remains unclear how the observed links should be interpreted. It is open to question if students' emotions impact their learning, if success and failure at learning influence the development of their emotions, if other variables cause the association, or if several of these possibilities are at work. Given the need to acquire knowledge about the antecedents of both students' achievement and their emotions, this is an issue of considerable theoretical and practical importance. To address this issue, the present investigation went beyond merely observing correlations at a single point in time and attempted to disentangle the causal ordering of these constructs across multiple waves of data collection and a developmental time span of several school years.

The investigation is based on a reciprocal effects model of emotion and achievement that posits that the two variables reciprocally influence each other over time. This stands in contrast to traditional unidirectional perspectives, which suggest that the link between emotion and achievement is simply due to effects of emotions on students' learning and performance. For example, correlations between test

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Reciprocal relations between students' academic enjoyment, boredom, and achievement over time

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ABSTRACT

The control-value theory (CVT) proposes that achievement emotions and academic achievement show reciprocal effects over time. Previous studies have examined how achievement emotions predict subsequent achievement. However, evidence is limited for whether achievement can also predict achievement emotions. To examine these reciprocal relations, data were collected about two achievement emotions: enjoyment and boredom, and mathematics achievement over four waves in a single school year in primary school students in Years 5 and 6. Results from structural equation modeling supported reciprocal relations between emotions and achievement. Higher enjoyment and lower boredom predicted greater subsequent achievement and, in turn, greater academic achievement predicted subsequent greater enjoyment and lower boredom. Furthermore, the relations between emotions over time were mediated by achievement. These findings build on the evidence base for CVT and further understanding of relations between achievement emotions and academic achievement in younger students.

1. Introduction

The aim of this study was to further understand the relationship between academic emotions and academic achievement in children. A number of studies have shown that emotions are not merely a by-product of learning and achievement but, critically, impact on subsequent achievement in tandem with self-system

Frenzel, Goetz, & Perry, 2007; Pekrun, Goetz, Titz, & Perry, 2002). In CVT, achievement emotions are positioned as an antecedent of academic achievement. However, relations between achievement emotions and academic achievement are not unidirectional. Academic achievements are also thought to influence subsequent emotions. Hence relations are expected to be reciprocal; academic emotions predict subsequent academic achievement and vice

- Enjoyment en boredom
- “Mathematics achievement positively predicted enjoyment and negatively predicted boredom.”
- “Enjoyment positively, and boredom negatively, predicted subsequent mathematics achievement.”

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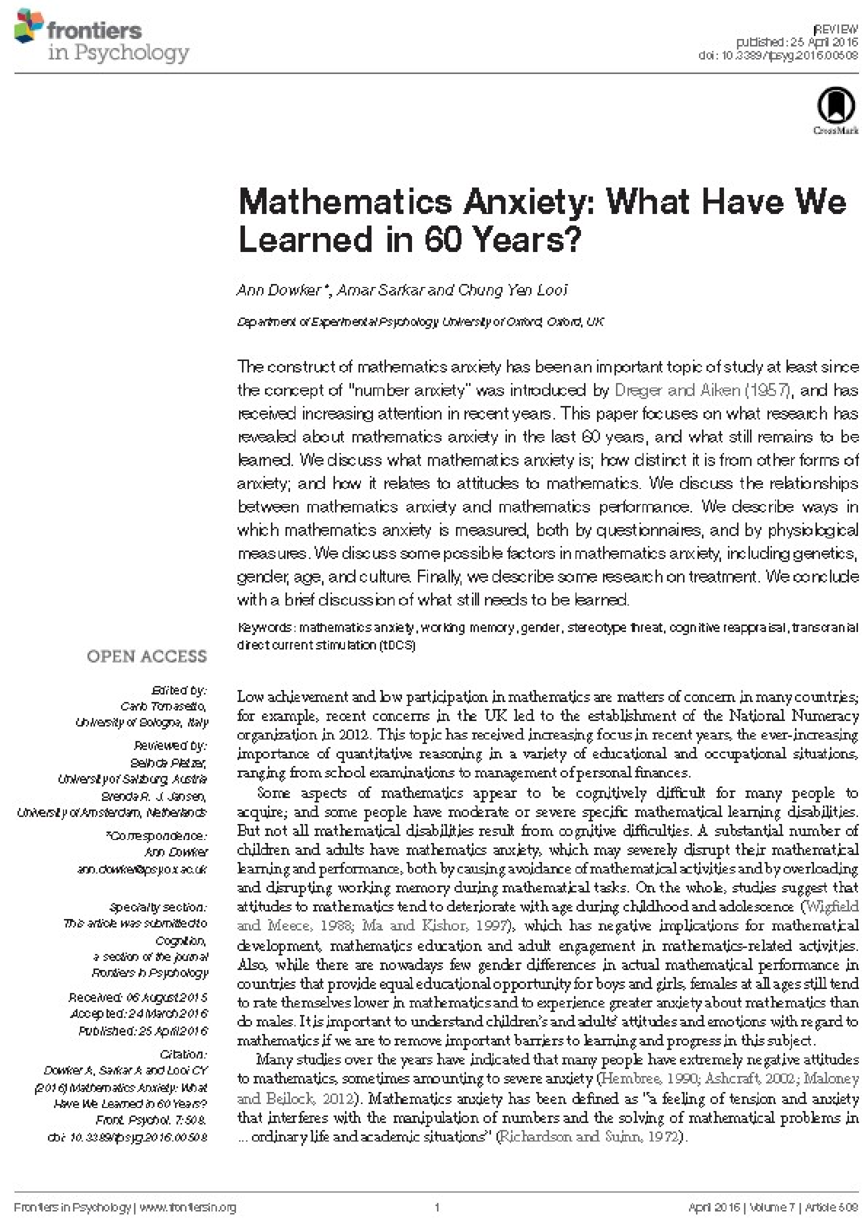
- Mathematics anxiety
- Dat is geen motivatie, maar beinvloedt prestatie.
- “the relationship between mathematics anxiety and mathematics self-concept is reciprocal: each influences the other.”
- Rol ouders.



frontiers in Psychology

REVIEW
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Mathematics Anxiety: What Have We Learned in 60 Years?

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The construct of mathematics anxiety has been an important topic of study at least since the concept of "number anxiety" was introduced by Dreger and Aiken (1957), and has received increasing attention in recent years. This paper focuses on what research has revealed about mathematics anxiety in the last 60 years, and what still remains to be learned. We discuss what mathematics anxiety is; how distinct it is from other forms of anxiety; and how it relates to attitudes to mathematics. We discuss the relationships between mathematics anxiety and mathematics performance. We describe ways in which mathematics anxiety is measured, both by questionnaires, and by physiological measures. We discuss some possible factors in mathematics anxiety, including genetics, gender, age, and culture. Finally, we describe some research on treatment. We conclude with a brief discussion of what still needs to be learned.

Keywords: mathematics anxiety, working memory, gender, stereotype threat, cognitive reappraisal, transcranial direct current stimulation (tDCS)

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Low achievement and low participation in mathematics are matters of concern in many countries; for example, recent concerns in the UK led to the establishment of the National Numeracy organization in 2012. This topic has received increasing focus in recent years, the ever-increasing importance of quantitative reasoning in a variety of educational and occupational situations, ranging from school examinations to management of personal finances.

Some aspects of mathematics appear to be cognitively difficult for many people to acquire; and some people have moderate or severe specific mathematical learning disabilities. But not all mathematical disabilities result from cognitive difficulties. A substantial number of children and adults have mathematics anxiety, which may severely disrupt their mathematical learning and performance, both by causing avoidance of mathematical activities and by overloading and disrupting working memory during mathematical tasks. On the whole, studies suggest that attitudes to mathematics tend to deteriorate with age during childhood and adolescence (Wigfield and Meece, 1988; Ma and Kishor, 1997), which has negative implications for mathematical development, mathematics education and adult engagement in mathematics-related activities. Also, while there are nowadays few gender differences in actual mathematical performance in countries that provide equal educational opportunity for boys and girls, females at all ages still tend to rate themselves lower in mathematics and to experience greater anxiety about mathematics than do males. It is important to understand children's and adults' attitudes and emotions with regard to mathematics if we are to remove important barriers to learning and progress in this subject.

Many studies over the years have indicated that many people have extremely negative attitudes to mathematics, sometimes amounting to severe anxiety (Hembree, 1990; Ashcraft, 2002; Maloney and Beilock, 2012). Mathematics anxiety has been defined as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in ... ordinary life and academic situations" (Richardson and Suinn, 1972).

Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years?. *Frontiers in psychology*, *7*, 508.

THEMA: BIJWERKINGEN

- 80s werk Sweller (Cognitive Load Theory) is interessant.
- Maar naar mijn mening niet altijd goed ‘vertaald’?
- Daarom ga ik over dit interessante artikel uit 1982 vertellen. Zoals wel vaker in de 80er jaren is de sample size klein.
- Volgens Sweller (2016) staan deze experimenten aan de basis van de theorie.

Sweller, J. (2016). Story of a research program. *Education Review*, *23*.

THEMA: BIJWERKINGEN

Bewerkingen x3 -69

De a in 2a staat voor ‘alternating’

	Start	Doel	Oplossing								
2a	60	111	x3	-69							
4a (1)	31	3	x3	-69	x3	-69					
4a (2)	81	453	x3	-69	x3	-69					
6a	34	21	x3	-69	x3	-69	x3	-69			
10a	35	156	x3	-69	x3	-69	x3	-69	x3	-69	
2c	35	315	x3	x3							

Twee groepen: 1 groep ‘probeer maar wat’, 1 groep werd aan oplossingen herinnerd.

Bijvoorbeeld na het oplossen van 6a:

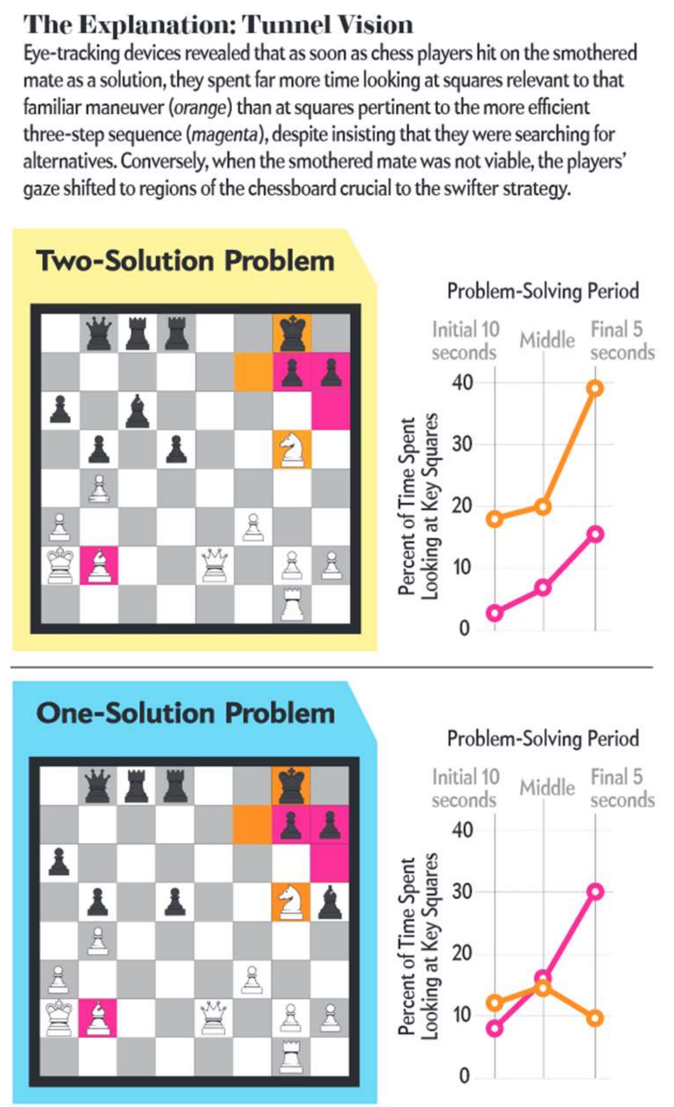
- De oplossing van het eerste problem was x –
- De oplossing van het tweede problem was x – x –
- De oplossing van het derde problem was x – x –
- De oplossing van het vierde problem was x – x – x –

Daarna werd gevraagd om de laatste oplossing te onthouden, en nog een keer in te voeren.

Einstellung

- “Often called a problem solving set, Einstellung refers to a person's predisposition to solve a given problem in a specific manner even though better or more appropriate methods of solving the problem exist.”
- Einstellung komt vaker voor
- Algebra
- Schaken

- Ergo…balans nodig….laten zien niet genoeg



https://www.sciencedirect.com/science/article/pii/S0010027708001133

Kortom, de claims zijn het **STARTPUNT**, niet het **EINDPUNT**

AFRONDING

Wat hebben we gezien

- Je moet dieper graven om een completer beeld te zien.
- Uitspraken zijn het startpunt, niet het eindpunt.
- Onderzoek heeft veel inzichten te bieden, maar lees breed, en bevraag de context.
- Onderzoek begrijpen betekent vaak meer vragen dan antwoorden. Maar het draagt desondanks bij aan inzichten.

Bedankt

- Vragen?
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