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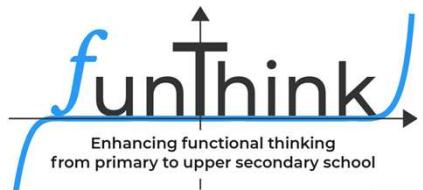
The word "funThink" is written in a large, bold, sans-serif font. The letter "f" is blue and curves upwards and to the right. The letters "unThink" are dark grey and positioned below the "f". A thick black horizontal arrow starts at the bottom left, passes through the "un" and "Think" parts, and ends at the top right. A thin black vertical arrow points upwards from the center of the "un" part.

Enhancing functional thinking
from primary to upper secondary school

|

Mara Otten, Bartjan Vollmuller & Michiel Veldhuis

*FunThink - Erasmus+ Strategic Partnership Multiplier event
op de Panama-conferentie 2022*



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- Ludwigsburg University of Education (Duitsland)



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- Universiteit Utrecht, Utrecht (Nederland)

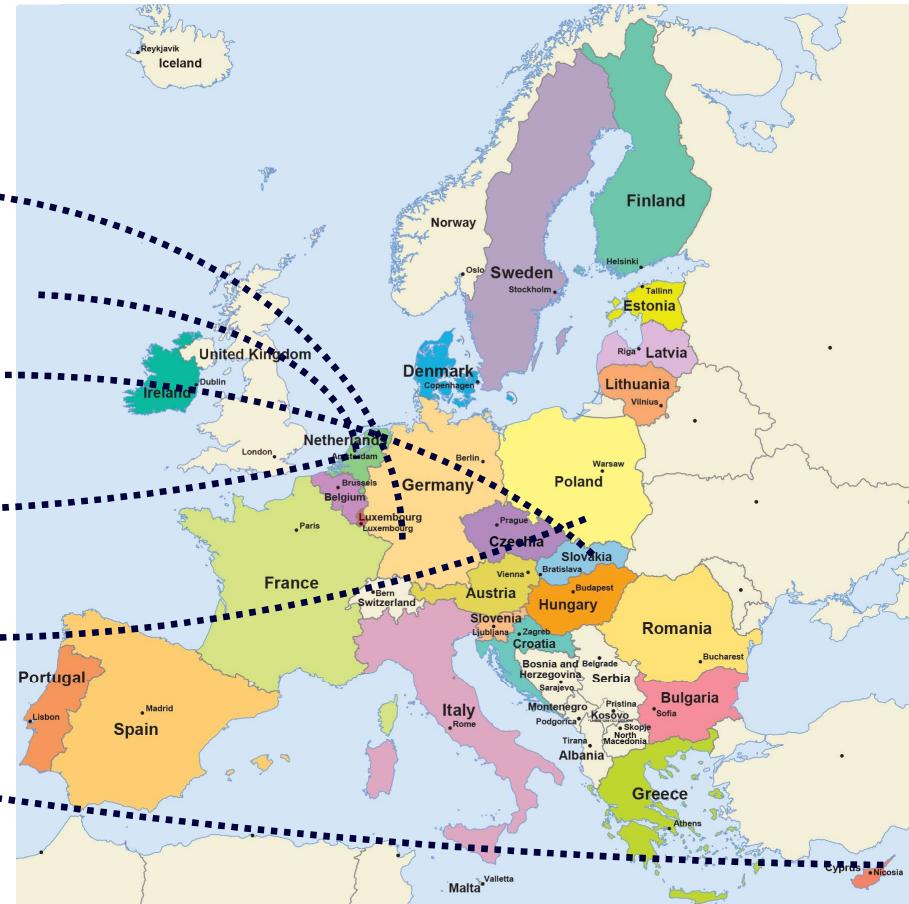


- Pedagogical University of Krakow (Polen)



- University of Cyprus (Cyprus)

<http://funthink.eu/partners-team.html>



HOGESCHOOL
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Ga naar www.menti.com en gebruik code 7723 2255

Wat zou functioneel denken kunnen zijn?



Functioneel denken

eigen gedachten
samenvangen van verschillende gedachten
samengang leren inzien
dagelijkse dingen
info filteren
dagelijks leven
nadenken
reden hebben
toepassen
denken in oplossingen
computational thinking
samenvangen van verschillende gedachten
ict
functioneel voor het leven
denken
betekenisvol
doel voor ogen
geïntegreerd
probleemoplossen
flexibel
praktisch
praktisch denken
niveau van denken
dat richt zich op hoger

Nederland

Functional thinking



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an appropriate way to int
relationship between
thinking functionally
algebra thinking seem like
as syaura **algebraic thinking**
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teacher
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defahayu math teacher
a one way thingking
generalizing about
thingking funtionally
function have a lot student
algebra thinking

Indonesia

"a way of thinking in terms of relationships, interdependencies, and change"
"the process of building, describing, and reasoning with and about functions"



Wat gaan we het komende uur doen?

- FunThink
- Wat is functioneel denken?
- Hoe wordt functioneel denken onderwezen in Europa?
- Ontwerpprincipes voor functioneel denken
- Voorbeelden van activiteiten
- Reflectie op activiteiten

2020-2021

Modules voor
het
ontwikkelen
van functioneel
denken van
leerlingen

2021-2022

Nascholing
leraren voor
het
ondersteunen
van de
ontwikkeling
van functioneel
denken

2022-2023

Open
educatief,
interactief en
multilinguaal
digitaal
platform voor
functioneel
denken



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2020-2021

Modules voor
het
ontwikkelen
van functioneel
denken van
leerlingen

- Visiedocument:
 - Literatuurreview (+/- 70 artikelen)
 - Interviewstudie (+/- 35 educators)
 - Nationale context (+/- 5 landen)

- Ontwikkelen leeromgevingen



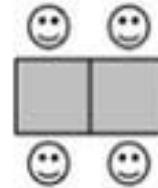
Een voorbeeld¹

Brady is celebrating his birthday at school. He wants to make sure he has a seat for everyone. He has square desks.

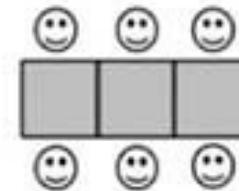
He can seat 2 people at one desk in the following way:



If he joins another desk to the first one, he can seat 4 people:



If he joins another desk to the second one, he can seat 6 people:



¹ Stephens et al. (2017)



Een voorbeeld¹

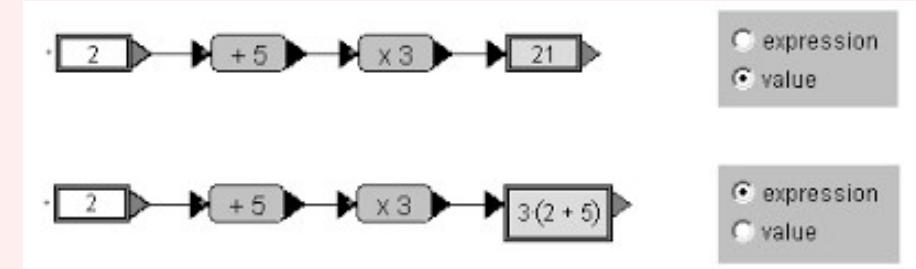
Niveaus:

- Recursief – “the number of people goes up two at a time”
- Covariationeel – “every new desk you add two people”
- Correspondentie – “times two”
- Functioneel – “by multiplying the desks by two you get the number of people”

¹ Stephens et al. (2017)

Vier aspecten van functies²

- Input-output

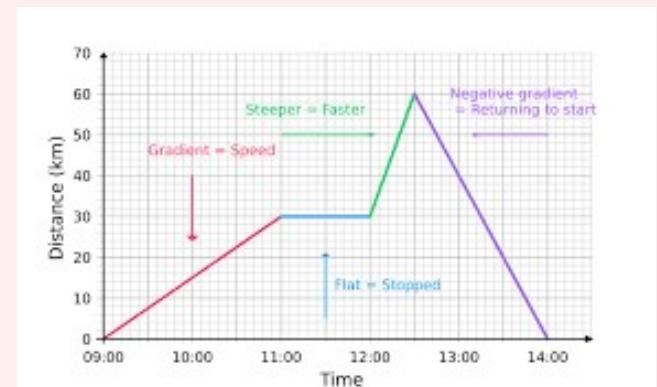


² Pitallis et al. (2020) & Doorman et al. (2015).



Vier aspecten van functies²

- Input-output
- Covariatie

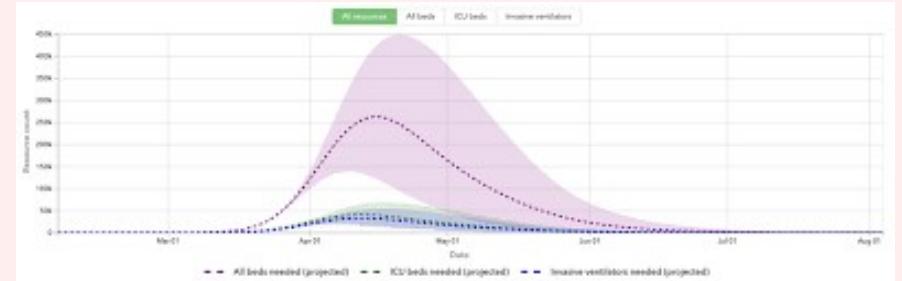


² Pitallis et al. (2020) & Doorman et al. (2015).



Vier aspecten van functies²

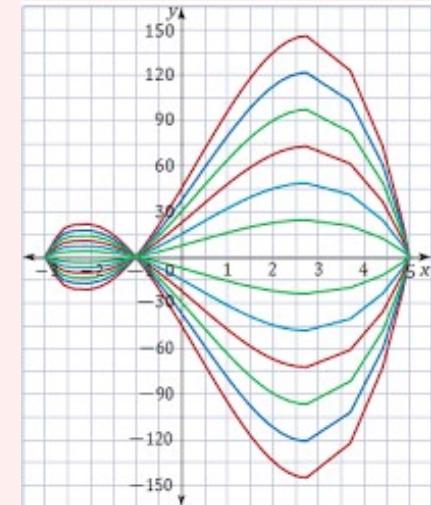
- Input-output
- Covariatie
- Correspondentie



² Pitallis et al. (2020) & Doorman et al. (2015).

Vier aspecten van functies²

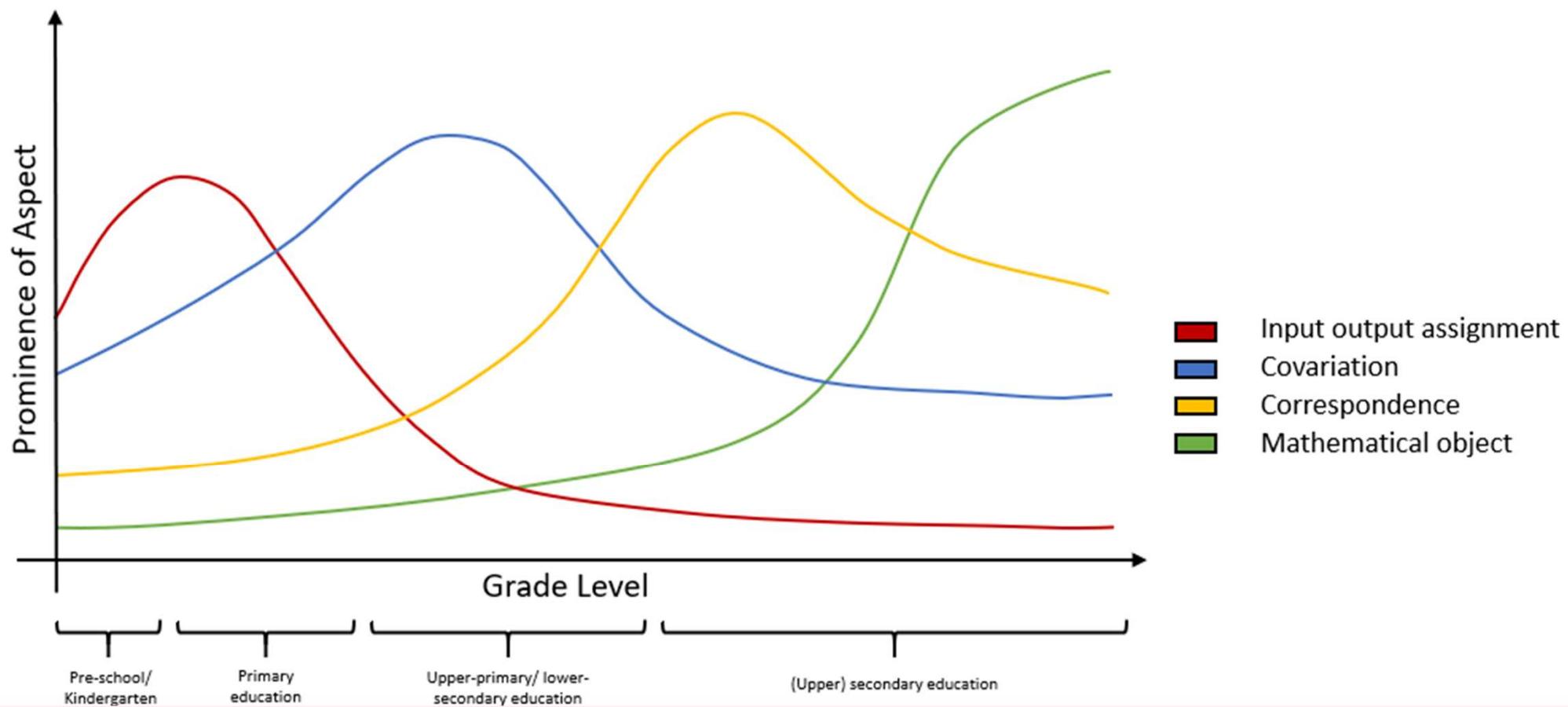
- Input-output
- Covariatie
- Correspondentie
- Wiskundig object



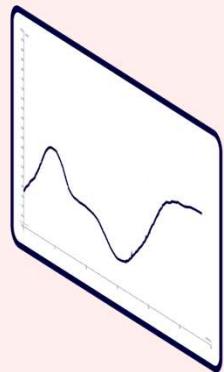
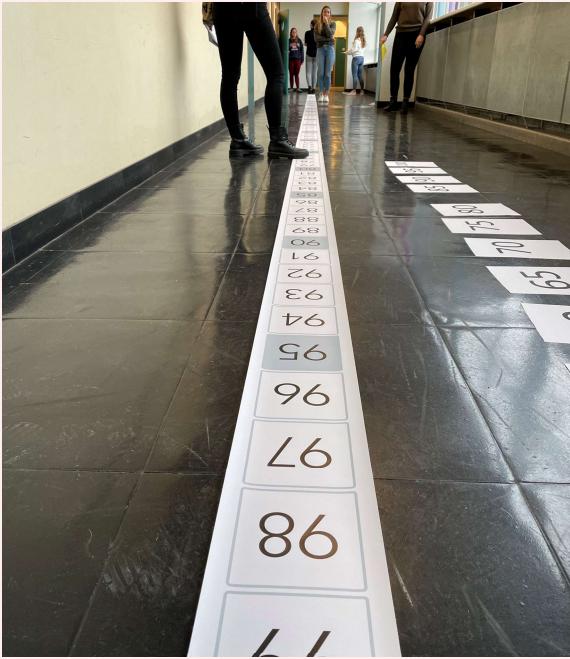
² Pitallis et al. (2020) & Doorman et al. (2015).

National situations

School-type	Grade	Germany			Netherlands			Poland		
Kinder-garten		general problem solving problem solving with numbers			general problem solving problem solving with numbers			general problem solving		
Primary School	1 (5 years)	number addition number subtraction number multiplication number division number fractions			number addition number subtraction number multiplication number division number fractions			number problem solving		
	2 (7 years)									
	3 (8 years)									
	4 (9 years)									
	5 (10 years)									
	6 (11 years)									
	7 (12 years)									
Secondary School	8 (13 years)	linear functions non-linear functions quadratic functions exponential functions logarithmic functions absolute value functions			proportional relationships linear functions quadratic functions exponential functions logarithmic functions absolute value functions			proportional relationships linear functions quadratic functions exponential functions logarithmic functions absolute value functions		
	9 (14 years)	quadratic functions			exponential functions logarithmic functions general properties of a function			quadratic functions exponential functions logarithmic functions absolute value functions root functions basic transformations applied on functions		
	10 (15 years)	quadratic functions exponential functions logarithmic functions absolute value functions root functions basic transformations applied on functions						quadratic functions exponential functions logarithmic functions absolute value functions root functions basic transformations applied on functions		
	11 (16 years)	universal functions linear functions non-linear functions quadratic functions exponential functions logarithmic functions absolute value functions root functions basic transformations applied on functions			algebraic functions transformations			calculus probability statistics		
	12 (17 years)									
	13 (18 years)									



Aan de slag!





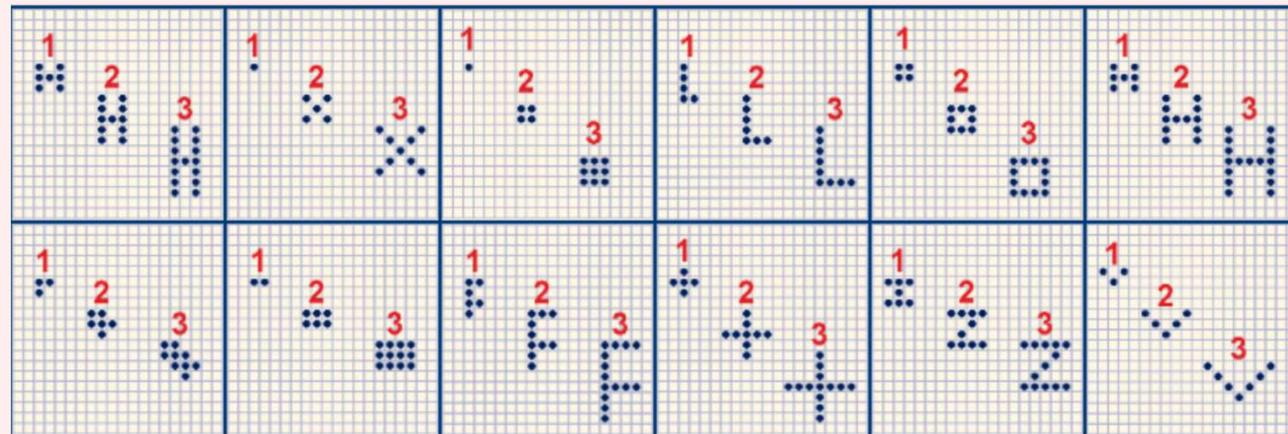
Ontwerpprincipes voor functioneel denken

- **Onderzoekend leren**
- **Embodiment**
- **Betekenisvol**
- **Digitale tools**



Ontwerpprincipes voor functioneel denken

- Onderzoekend leren



Ontwerpprincipes voor functioneel denken

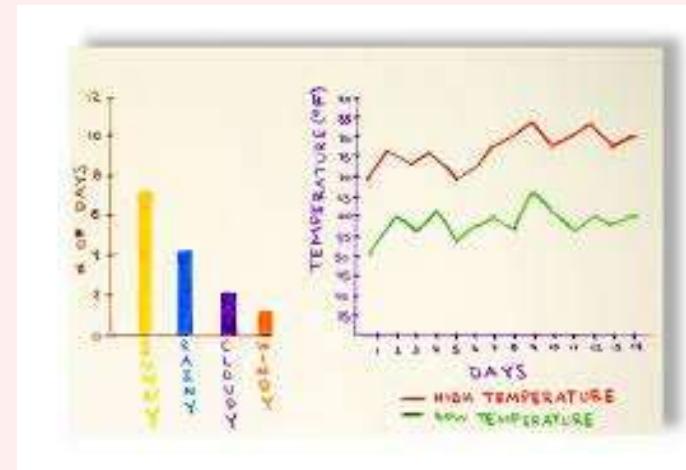
- Onderzoekend leren
- Embodiment





Ontwerpprincipes voor functioneel denken

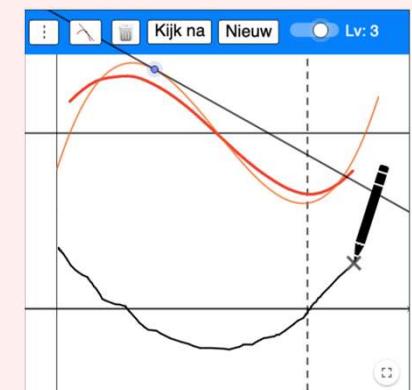
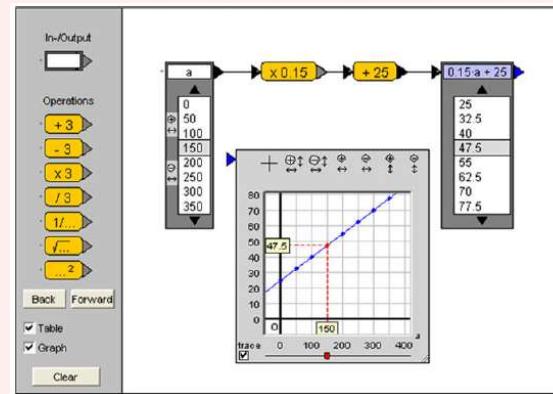
- Onderzoekend leren
- Embodiment
- Betekenisvol

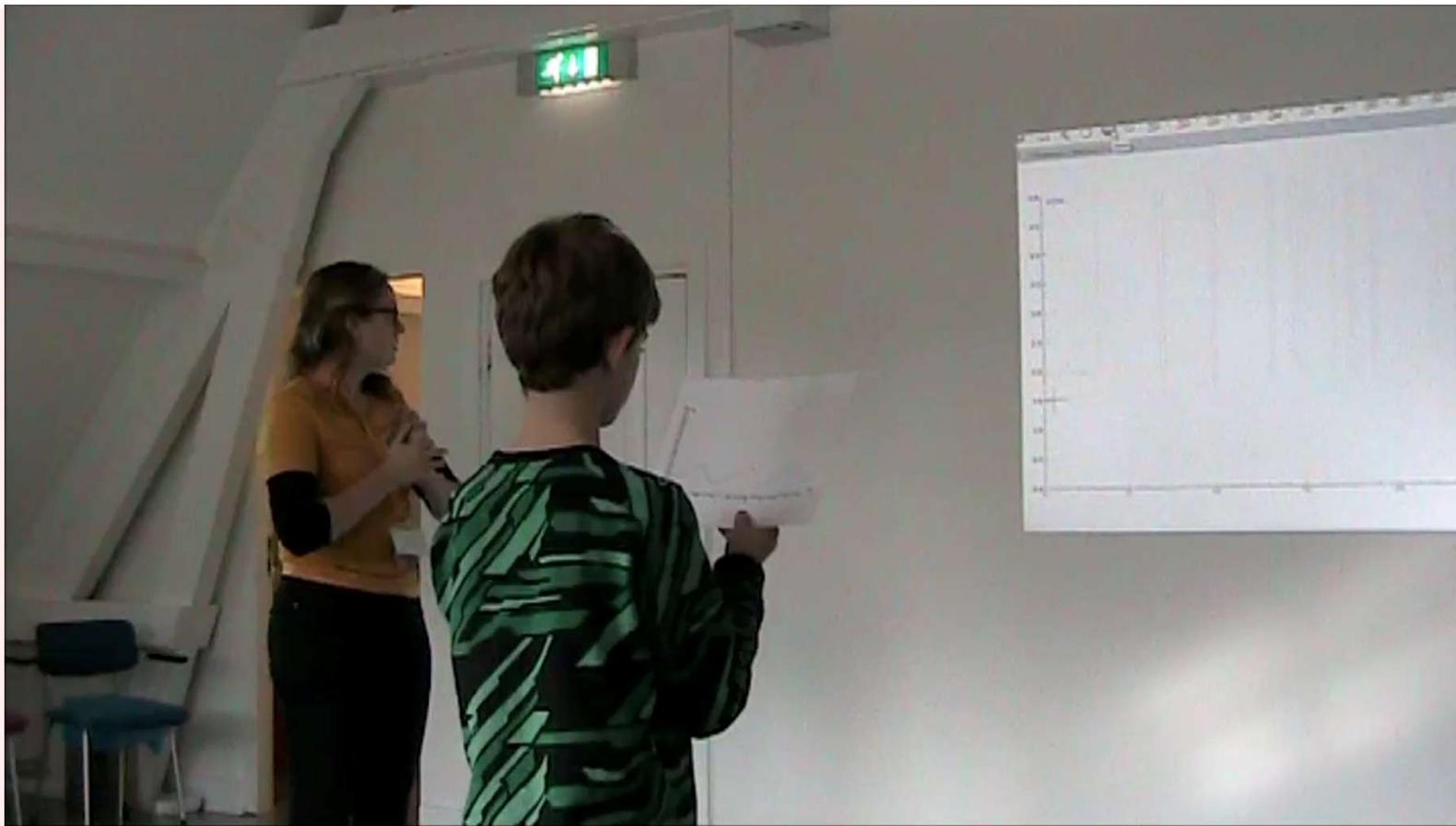




Ontwerpprincipes voor functioneel denken

- Onderzoekend leren
- Embodiment
- Betekenisvol
- Digitale tools



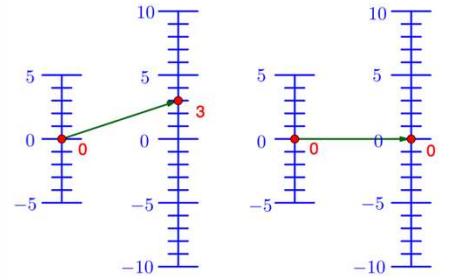


Duijzer et al., 2019





<https://www.geogebra.org/classic/fqema5wm>



Groep 6-8

2. Zoe is creating the following pattern.
 (a) How many squares would Figure 4 have?
 (b) How many more squares would each next figure have?

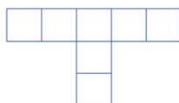
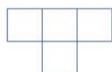
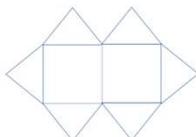
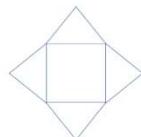


Figure 1

Figure 2

3. A pattern begins with number 5 and repeatedly adds 3. What are the pattern?

4. Kai is constructing the following pattern.



2. Match the situation with the corresponding graph

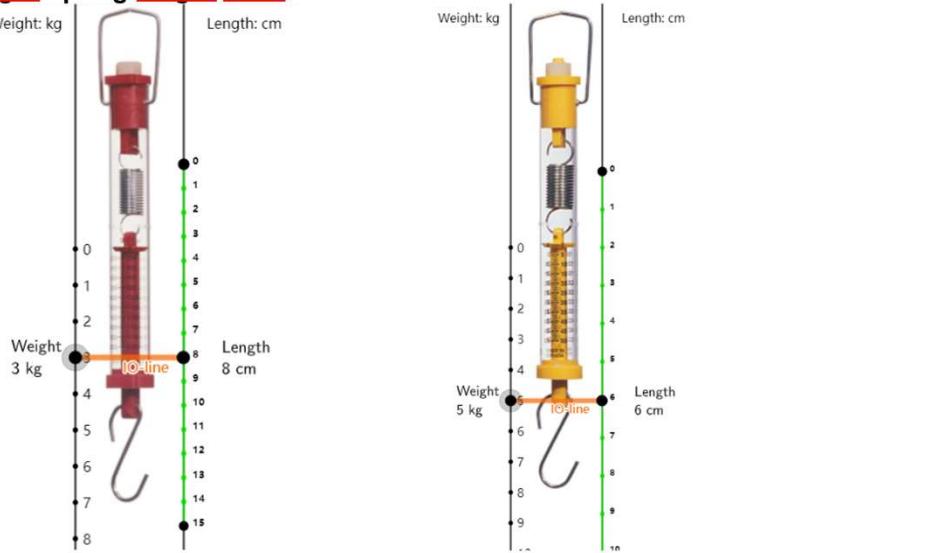
Description	Graph
A plumber charges with the following plan: A flat cost of €20 for a service call and an additional charge of €30 per hour	
At a gas station the price of petrol is €1,38 per litre	
An offer for a birthday party charges €14 per guest. In the total cost a discount of €40 is applied.	

<https://gizmos.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1035>



Klas 1-2

2. Introduce the IO-line, which can help students better understand the relationship between input and output values. Let students fill out the object weight-spring length table.



- We tell students that they will start by cutting A3 paper at the beginning of the lesson. "You will cut the paper into two approximately equal parts. Then you put these pieces on top of each other and cut them again (short video on how to cut the paper and how many pieces they get). You will continue cutting for approximately 10 minutes. Before you start cutting, estimate how many pieces of paper you will get after ten cuts and write your estimation in the activity sheet."

- Students are cutting and filling the table:

Number of cuttings	0	1	2	...	10
Number of pieces					

- Imagine you are plotting a graph from this table with 1 cm to one unit on each axis. How far along the x-axis could you go before the graph reached the top of a sheet of A4 (A3) paper?

How far along the x-axis would you have to go so that the graph was tall enough to reach the highest peak in our country (our capital city, the Moon, the Sun)?

- The formula for the investigated function is $y=2^x$. Create the table for the function, which formula looks similar: a) $y=x^2$ and b) $y=2x$
a)

x	0	1	2	...		10
$y=x^2$						



- Each student sorts 4 groups of equations according to their solutions from the equation with the smallest solution to the equation with the biggest solution. Equations are in the frame which changes from red to green. It is green if the order is proper.

$\frac{1}{2}x + 139 = 150$ $\frac{1}{2}x + 139 = -2$ $\frac{1}{2}x + 139 = 93$ $\frac{1}{2}x + 139 = 518$	$2x - 308 = 0$ $2x - 308 = -99$ $2x - 308 = 107$ $2x - 308 = 236$
$-3x - 17 = -91$ $-3x - 17 = 155$ $-3x - 17 = \frac{3}{4}$ $-3x - 17 = 0$	$139 - \frac{1}{2}x = 141$ $139 - \frac{1}{2}x = \frac{1}{5}$ $139 - \frac{1}{2}x = -2$ $139 - \frac{1}{2}x = -\frac{1}{3}$

Klas 3-4

Use the blue points to control the arrow. Try to feel, what the right position is. You can display correct arrows you draw with the 'arrow button' this can also be done in 'real-time'. With the "1" button you can display the numbers along the nomogram.

The interface shows a nomogram for the function $f(x) = -x$. The graph features a horizontal axis with two blue circular markers at the ends and a vertical axis. A black line passes through the origin (0,0) with a negative slope. Two green circular points are placed on the line, one near the origin and one further down the line. To the left of the graph is a green vertical bar containing three buttons: an arrow pointing right, a button labeled "1", and a button with a circle icon. Below the graph, the URL <https://wiswise.nl/sensonomo/> is displayed.



New Check Easy Normal

X A **X B** **X C** **X D**

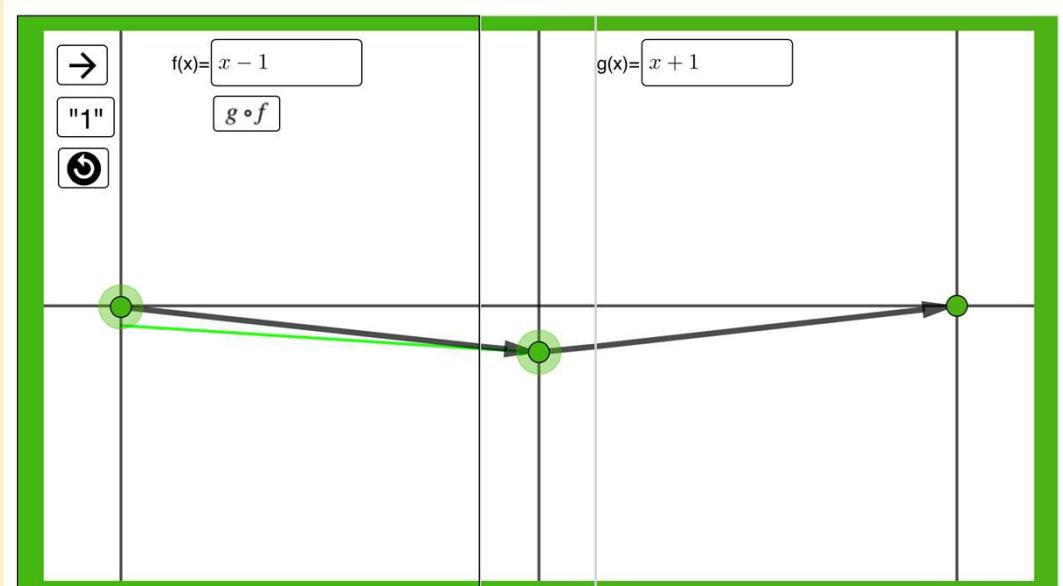
Below the arrows is a 4x5 grid of colored dots:

•	○	○	○	•
•	○	○	○	○
•	○	○	○	•
○	○	○	○	•

Check button

<https://www.geogebra.org/m/szgbfwgq>

Klas 5-6



<https://wiswise.nl/sensonomodouble/>



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Dank!

Voor vragen, opmerkingen, meer informatie
www.funthink.eu

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