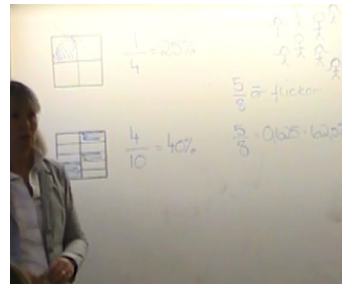


Using variation theory as a guiding principle of didactical design. With the focus on the object of learning, its critical aspects and how they can be taught.

Ulla Runesson  
University of Jönköping, Sweden and  
University of the Witwatersrand, South Africa



$$\frac{1}{4}=25\%$$

$$\frac{4}{10}=40\%$$

$$\frac{5}{8}=0,625=62,5\%$$

$$5\% \text{ of } 800=$$

$$6\% \text{ of } 700=$$

Lesson 1

$$\frac{1}{4}=25\%$$

$$\frac{1}{2}=50\%$$

$$\frac{1}{5}=20\%$$

$$\frac{1}{100}=1\%$$

$$1=100\%$$

$$25\% \text{ of } 200=$$

$$10\% \text{ of } 200=$$

$$40\% \text{ of } 200=$$

$$75\% \text{ of } 200=$$

Lesson 2

Different theories explain learning in different ways

Emphasizes e.g.

- intellectual maturity,
- function of the brain,
- activity,
- cognitive change,
- the social and linguistic dimensions,

Variation theory (Marton & Booth, 1997; Marton, Runesson & Tsui, 2004; Lo, 2012; Marton, in press)

When we learn, we always learn *something*.  
Learning has an object.  
The object of learning  
For every object of learning there must be something learned

Themes of my presentation:

- How are differences in learning outcomes explained by variation theory?,
- Learners' meaning making and the nature of 'the what',
- How can learning be designed from the point of view of variation. Does the pattern of variation matter?

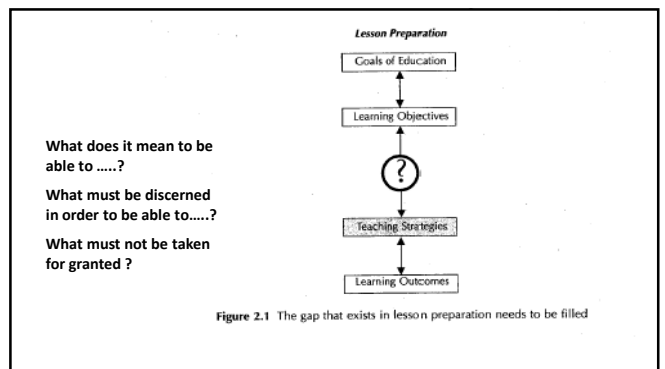
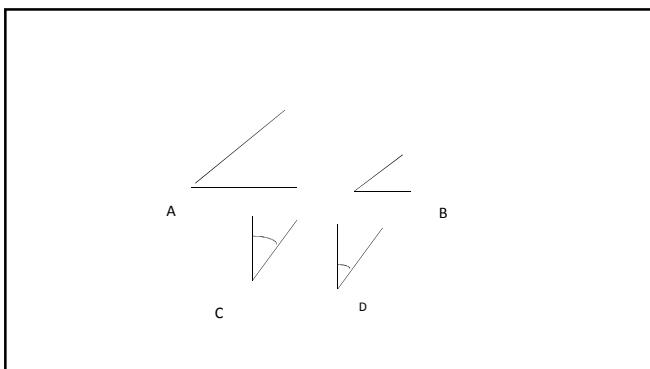
Splitting 9 marbles into two boxes (Neuman, 1987; 2013)

6 and 6	4 and 9
5 and 5	5 and 9
4 and 4	3 and 9
10 and 10	7 and 9
8 and 8	

The cardinal aspect is attended to                      The ordinal aspect is attended to

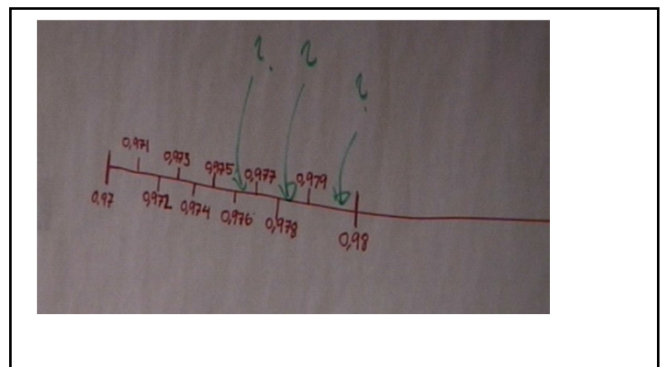
Learning simple arithmetic...

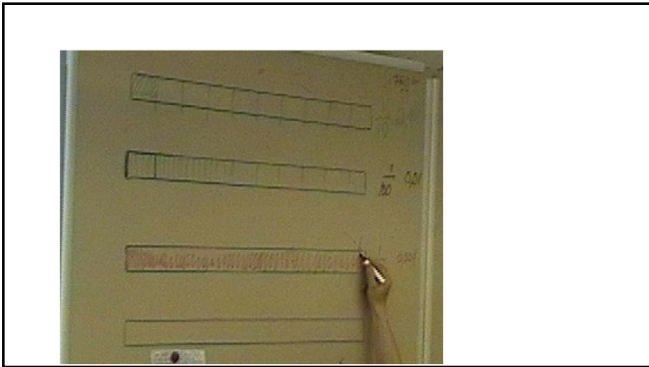
...takes the *simultaneous* discernment of the ordinal, cardinal and part-part-whole aspect (critical aspects)



Results before and after the lesson (Kullberg, 2010)

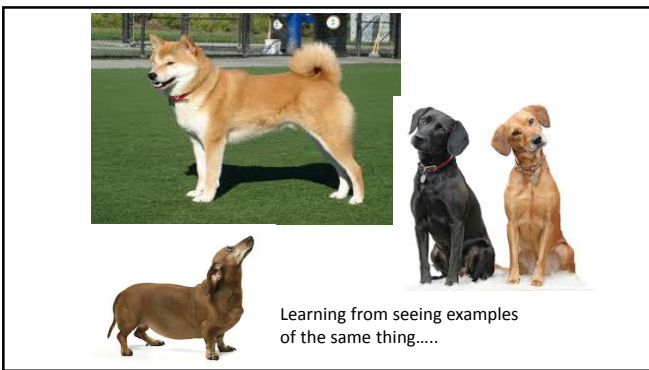
	Class A N=19	Class B N=17
Are there decimal numbers between 0,97 and 0,98? <b>Pre-test</b> Many numbers / endless	<b>5% (1)</b>	<b>24% (4)</b>
One number	21% (4)	29% (5)
Ten numbers	0% (0)	0% (0)
No numbers	42% (8)	18% (3)
Other	32% (6)	29% (5)
Are there decimal numbers between 0,97 and 0,98? <b>Post-test</b> Many numbers / endless	<b>21% (4)</b>	<b>94% (16)</b>
One number	0% (0)	0% (0)
Ten numbers	47% (9)	6% (1)
No numbers	21% (4)	0% (0)
Other	11% (2)	0% (0)





What aspects were the critical aspects?

- The interchangeable representation
- The number as a part of a whole
- The divisibility of parts



...or by contrasting? Seeing what is *not* a dog?



We learn by the experiencing of differences rather than of similarities.

Lesson 1

$\frac{1}{4} = 25\%$   
 $\frac{4}{10} = 40\%$   
 $\frac{5}{8} = 0,625 = 62,5\%$   
 5% of 800=  
 6% of 700=

$\frac{1}{4} = 25\%$   
 $\frac{1}{2} = 50\%$   
 $\frac{1}{5} = 20\%$   
 $\frac{1}{100} = 1\%$   
 $1 = 100\%$

25% of 200=  
 10% of 200=  
 40% of 200=  
 75% of 200=

Lesson 2

1.  $y = 2x$

2.  $y = \frac{1}{2}x$

3.  $f(x) = \frac{x}{2}$

4.  $p(x) = x^2$

5.  $h(x) = \frac{2}{x}$

6.  $g(x) = 2^x$

The example space

(Pillay, manuscript)

## Lesson 1

Eqn	x	-4	-2	-1	0	1	2	3	4	Type of graph	
a	$y = 2x$	-8	-6	-4	-2	0	2	4	6	8	Linear
b	$y = \frac{1}{2}x$	-2	-1	0	1	2	3	4	5	6	Linear
c	$y(x) = \frac{x}{2}$	-2	-1	0	1	2	3	4	5	6	Linear
d	$h(x) = \frac{2}{x}$	-1	-2	-4	-8	-16	-32	-64	-128	-256	Hyperbola
e	$k(x) = x^2$	16	4	1	0	1	4	9	16	25	quadratic
f	$m(x) = 2^x$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	Exponential

Example graph → substituting values → plotting → drawing the

## Lesson 2

$y = 2x$  and  $y = \frac{1}{2}x$

$y = \frac{1}{2}x$  and  $f(x) = \frac{x}{2}$

$f(x) = \frac{x}{2}$  and  $h(x) = \frac{2}{x}$

$p(x) = x^2$  and  $g(x) = 2^x$

Comparing pairs of functions

## Principles of variation theory

- Learning implies seeing something in a new way
- Experiencing something in a certain way takes the simultaneous discernment of certain aspects.
- For every object of learning there are certain aspect that must be discerned
- The discernment of an aspect presupposes an experienced variation of that particular aspect. (Marton, Runesson, & Tsui, 2004)

Thank you!

## Further readings

- Lo, M. L. (2012). Variation theory and the improvement of teaching and learning [https://gupea.ub.gu.se/bitstream/2077/29645/5/gupea\\_2077\\_29645\\_5.pdf](https://gupea.ub.gu.se/bitstream/2077/29645/5/gupea_2077_29645_5.pdf)
- Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahawa NJ: Erlbaum.
- Kullberg, A., Runesson, U., Mårtensson, P. (2013). The same task? - different learning possibilities? In C. Margolinas (Ed.) *Task Design in Mathematics Education Proceedings of ICMI Study 22*, ISBN 978-2-7466-6554-5 <http://hal.archives-ouvertes.fr/hal-00834054>